Aristotle

Prior Analytics

Translated by
Robin Smith

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translated, with introduction,
notes, and commentary, by

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Contents

NOTES TO BOOK A

Chapter 1

24a10–15. 'Demonstration' (*apodeixis*) is the subject of the first Book of the *Posterior Analytics*; in the *Prior*, especially Book A, the attention is instead on 'deduction' (*sullogismos*). However, at the beginning of A 4 (25b26–31), Aristotle tells us that since a demonstration is a species of deduction, an account of the former should first treat the latter. The entire *Analytics* would then form a single treatise on demonstration, with the *Prior* serving as a prelude to the *Posterior*. This traditional view of the relationship of the parts of the *Analytics* is in my view confirmed by the internal structure of the work, and by the fact that the only treatise in the entire Aristotelian *corpus* which makes any substantive use of the results of the *Prior Analytics* is the *Posterior*.

The Greek commentators find a puzzle in the grammar of the first sentence, which appears to ask two questions: (1) what is his inquiry *about* (*peri ti*), and (2) what is it *of* (*tinos*). Since the pronoun is in the accusative case in the first instance and in the genitive case in the second, these are answered by phrases marked only by their case endings: (1) <about> demonstration (accusative case), (2) <of> demonstrative science (genitive case). The question what faculty an area of inquiry falls under would be an unusual one for Aristotle to raise: the only remotely comparable cases are the discussions in *Metaphysics* III, VI, VII, concerning whether there is such a science as first philosophy. Alexander, who develops this line of interpretation (9.17–23), suggests that the ability with which we understand demonstration in general is the same as the ability with which we understand demonstrations in particular cases. But whatever may be the merits of this approach, I think it goes beyond what we can reasonably be sure of. My translation tries to remain non-committal. See Brunschwig *1981* for a persuasive defense of another interpretation.

24a11. 'science': there are many reasons for objecting to this as a translation of *epistēmē*. I have stuck by it because all the alternatives have even more difficulties. (But the reader should regard it as a translator's term of art; its modern associations, while not wholly irrelevant to Aristotle, are to be resisted.) In ordinary Greek, *epistēmē* can be rendered comfortably as 'knowledge,' 'skill,' or sometimes 'art.' In Plato and Aristotle, it takes on more precise senses laden with epistemic distinctions. The translation 'understanding' has acquired some popularity in recent years (see Barnes *1975*, Burnyeat *1981*), but this is in my judgment even more awkward in some contexts (there is no adjective which stands to it as 'scientific' to 'science,' and it is forced to talk about, for instance, arithmetic as an 'understanding'). In a number of places, where convenient, I adopt the more expansive translation 'scientific understanding'; in others, when it occurs only as a part of an example, I make it 'knowledge.'

24a12. 'Deduction' translates *sullogismos*. The English word 'syllogism' ultimately derives, not simply from this Greek word, but from Aristotle's use of it in the *Prior Analytics*; somewhat paradoxically, it is for this reason a poor translation. The history of logic has created such a strong association between 'syllogism' and the particular forms of argument studied in A 4–22 that the modern reader cannot help being confused by its presence. Aristotle does not intend to *define* the word *sullogismos* to have a sense as narrow as 'syllogism' (see the notes on the definition in 24b18–22), nor was it his own coinage. Etymologically, a *sullogismos* is the result of an act of 'syllogizing' (*sullogizesthai*). The latter verb is the compound of *sun-* ('together') and *logizesthai* ('calculate'), so that the nontechnical meaning of *sullogizesthai* is 'reckon up' or 'compute' (a sense found as early as Herodotus), and a *sullogismos* is a 'computation.' Plato uses both *sullogismos* and *sullogizesthai*, sometimes in connection with drawing conclusions in an argument, though often with the broader sense of reckoning or calculating. I follow Corcoran's suggestion in translating it 'deduction.'

24a14. 'What we mean by': literally, 'what we call' (*ti legomen*). This is Aristotle's usual way to introduce the technical senses of terms.

24a16. 'sentence' (*logos*): this term has a range of meaning too broad to be captured with a uniform English rendering. It can also mean 'discourse' (e.g., an entire speech, or even something the length of the *Iliad*).

24a16–b15. The term *protasis*, translated here as 'premise,' is difficult to render in English without prejudice. It is not found before Aristotle at all, although he uses it in the *Topics* in a way that suggests that it was at least current in Academic circles and probably was not his coinage. Translators have favored two choices: 'proposition' and 'premise.' The first, if interpreted to mean a sentence with a truth value, would be most consonant with the definition Aristotle gives here; the second, which emphasizes a certain role in an argument, fits more naturally with the actual use of the term in the bulk of the *Prior Analytics*. Since argumentative role is so often in question, I have elected to use 'premise,' even though this sometimes makes Aristotle talk about premises without arguments (though he never refers to the *conclusion* of an argument as a *protasis*). See also the Note on 42a38.

In fact, what has happened is that Aristotle's terminology has evolved over time. In the *Topics*, a *protasis* is defined as a type of question (I.10, 104a8–11), and the etymological connection with the root verb *proteinein* ('hold out,' 'put forward,' 'propose') is strong: in the context of question-and-answer argument, a *protasis* is what one offers for acceptance to one's opponent. However, Aristotle came to understand that contained in every such 'premise' there is (at least implicitly) a declarative sentence (or a pair consisting of a sentence and its negation: see the note below on 24a25). He also developed a theory according to which every such sentence either affirms or denies one thing of one thing, so that a single assertion always contains a single subject and a single predicate. (In *On Interpretation*, he explains more complex sentences either as having complex subjects or predicates or as really equivalent to groups of sentences.) In the *Analytics*, logical form and argumentative role are not entirely separated; in *On Interpretation*, the separation is more complete, and Aristotle defines the expression 'declarative sentence' (*logos apophantikos*) in the way he here defines *protasis*. (See more on this below.)

Aristotle distinguishes three varieties of premise: *universal* (*kath'holou*: literally, 'of a whole'), *particular* (*en merei*, *kata meros*: literally, 'in part,' 'with respect to a part'), and *indeterminate* (*ahoristos*). The definition given here appeals to verbal markers (or their absence, in the case of indeterminates), but Aristotle does not rely exclusively on forms of expression (see, for instance, the remarks about the ambiguities of some of his modal expressions in the Notes on A 14, A 16, A 17). The locutions 'belongs to some,' etc., are peculiar to Aristotle and not at all everyday Greek. (See the Introduction for a brief discussion of Aristotle's language.)

24a20–21. 'without a universal or a particular': Aristotle has just defined 'universal' and 'particular' with certain phrases ('to every,' 'to some'). Thus, here 'a universal' or 'a particular' would just be an occurrence of some one of these phrases.

24a21–22. The two examples of indeterminate premises given here are actually quite different in form. 'Pleasure is not a good' is a straightforward categorical sentence with a simple subject and predicate but no indication of quantity. 'The science of contraries is the same' is more problematic. Aristotle uses this sentence, drawn from philosophical discussions of his time, more than once in the *Prior Analytics* (most extensively in B 26, 69b8–29; see also the Notes on 48b4–7, in which passage Aristotle seems to maintain that this very example is actually not a predication). Its meaning, more fully, is 'The science which has one of a pair of contraries as its object also has the other member of the pair as its object.' From a modern viewpoint, this has a somewhat complex structure: most contemporary logicians would see it as an assertion of the identity of two things each identified by a definite description. However, to judge by B 26, Aristotle sees it as having as its subject 'contraries' and as its predicate 'there is a single science of them' (see 69b8–9). So interpreted, it is presumably regarded here as indeterminate because it does not say 'of every <pair of> contraries' or 'of some <pair of> contraries.'

24a22–b15. This section reflects the complex background of the term *protasis*. Aristotle distinguishes *demonstrative* (*apodeiktikē*), *dialectical* and *deductive* (*sullogistikē*) premises. As in the definition of *protasis* itself, syntactical and pragmatic elements are combined in these distinctions. Underlying them all is the notion of a *contradiction* (*antiphasis*), that is, the contradictory pair consisting of an affirmation and the corresponding denial. Pragmatically, a demonstrative premise is the 'taking' (*lēpsis*) of one or the other member of such a pair (specifically, the 'true and primary' one), while a dialectical premise is the 'asking' (*erōtēsis*) of the contradictory pair itself: in effect, such a pair put as a question. The fundamental point is that the demonstrator and the dialectical arguer do different things: one asserts premises while the other gets them as answers to questions. According to the *Topics*, dialectical premises are questions which are 'proposed in the manner of a contradiction' (*kat' antiphasin proteinomena*), that is, admit a yes-or-no answer (see, e.g., *Top.* I.10, 104a14, 104a21, 104a26). The dialectician's skill includes the ability to take either reply and build an argument from it. Accordingly, in its dialectical origins *protasis* means 'question presented in the course of an argument.' By contrast, the demonstrator 'takes,' or assumes, premises as the basis for a proof. The critical difference between dialectical and demonstrative premises, then, is *how* the dialectician and the demonstrator respectively 'get' (*lambanein*) their premises (Alexander, 13.18–19, says that the difference is 'the way of taking'). In this passage, Aristotle isolates the element both practices have in common, which is 'one of the parts of the contradiction,' i.e., a simple affirmation or denial, and notes that even in the dialectical case, conclusions are drawn by 'taking something either to belong or not to belong'; the deductive premise is then defined in this neutral way. Apart from these argumentative roles, Aristotle also differentiates demonstrative and dialectical premises on semantic and epistemic grounds. Semantically, demonstrative premises must be true, though dialectical premises need not be. Epistemically, dialectical premises must be 'accepted' or 'respected' (*endoxos*) and 'apparent.' Although the present chapter does not describe the epistemic status of demonstrative premises, Aristotle discusses this at length in *Posterior Analytics* I.2 and briefly in *Topics* I.1 (100a27–b22). The notion of deductive premise here is the result of a distillation from all these contexts of a fundamental core meaning, excluding any epistemic properties; this is an important innovation of the *Prior Analytics*.

24a27. 'Something with respect to something' or 'something about something' (*ti kata tinos*) is a common phrase in Aristotle to indicate predication.

24b10–11. 'getting answers . . . deducing': the verb *punthanesthai*, which I translate 'getting answers,' means 'inquire,' 'learn through hearing.' Aristotle uses it in the logical works interchangeably with 'ask' (*erōtan*) and 'attack' (*epicheirein*) to apply to that participant in a formal dialectical exchange who asks questions in order to secure premises from which to refute the thesis which the other participant has undertaken to defend. This other participant is said to 'answer' (*apokrinesthai*) or to 'maintain' (*hupechein*).

24b12. 'in the *Topics*': I.10–11.

24b16–18. The last phrase of this definition, 'whether or not "is" or "is not" is added or divides them' (*ē prostithemenou ē dihairoumenou tou einai ē mē einai*) is an occasion of difficulty. Some interpreters, appealing to *Metaphysics* VI.4, suppose that Aristotle means that affirmative and negative statements respectively 'combine' (*suntithenai*) and 'divide' (*dihaireisthai*) their predicate and subject terms. But Aristotle does not elsewhere oppose *prostithenai* to *dihaireisthai*: he opposes *dihaireisthai* to *suntithenai* ('put together'), *sunhaptein* ('join together'), *sumplekesthai* ('weave together'), or *sunkeisthai* ('put together'), *dihairesis* to *sunthesis* or *sumplokē* (see *Met.* VI.4, IX.10 for a good sampling). *Prostithenai* always carries the suggestion of adding, rather than combining or joining; it is usually opposed to *aphairein*, which (as far as I know) is never used of the relation between predicate and subject in a denial. The grammar of the whole genitive-absolute phrase is also problematic. If we take the verbs as passive in sense, then it is unclear what it means to talk about 'being' or 'not being' being divided. If, in accordance with the attempt to associate the passage with the views in *Met.* VI.4, we try to associate *prostithemenou* with *einai* and *dihairoumenou* with *mē einai*, then we get something like "if 'being' adds or 'not being' divides," which (even apart from the fact that 'adds' and 'combines' are not synonyms) is grammatically implausible.

Ross takes Aristotle to be talking about the linguistic structure of a *protasis*, so that he only means to call attention to the copula (whether affirmative or negative) as something added to the two terms. If we follow his suggestion of striking out 'or dividing,' the phrase means 'when "is" or "is not" is added' and is then most plausibly taken as modifying 'predicated.' I think this is on the right track, but it would be better to avoid a textual change. The problem is to figure out just what Aristotle thinks of the copula. In fact, a copula is not an essential constituent of a predication in Greek, since one can accomplish the same result by simple concatenation, as in *pasa hēdonē agathon* ('every pleasure a good'), *hēdonē tis agathon* ('some pleasure a good'), to take two random examples from Aristotle (25a9, 25a11). If the copula is present, it might be positioned between subject and predicate, as in English, or added in front (e.g., *esti tis hēdonē agathon*, 'is some pleasure a good'). If the copula comes between the terms, we could plausibly describe it as 'dividing' them; if it is placed in front, we could say it has been 'juxtaposed' or 'added' (both acceptable senses for *prostithenai*). For that matter, in a predication without copula, we could say that one term is 'juxtaposed' or 'put next' (*prostithetai*) to the other. Now, Aristotle hardly ever uses *einai* to express a predication in the exposition of his theory of deductions (A 1–22), though it is somewhat more common later in the *Prior Analytics*. He might, therefore, want to say something at this point about the fact that 'is' and 'is not' may appear in premises in addition to the terms and the indicators of quantity. What he has just said is that a premise 'breaks up' (*dialuetai*) into terms, and thus, by implication, a premise is made up *only* of terms: what then do we say about 'is' and 'is not,' should they be present? Aristotle's response, I think, is that the copula is to be regarded as a purely optional occurrence: the premise still is composed of just its terms, whether or not a copula is present.

24b18–22. This celebrated definition appears in almost exactly the same form in *Topics* I.1, 100a25–27 (cf. *Sophistical Refutations* 1, 164b27–165a2, *Rhetoric* I.2, 1356b16–18). It has been the subject of enormous discussion by commentators ancient and modern, and I cannot summarize the range of these opinions here. (See the notes above on 24a12–13 for the history of the word *sullogismos.*) The definition is clearly intended to apply to a wide range of arguments: Aristotle does not regard *sullogismoi* as merely one species of valid argument. The exact range of the term is less certain, but it is surely less wide than 'valid argument' for most present-day logicians. Alexander points out that, according to the definition, the 'thing which results' (the conclusion) must be distinct from any of the 'things taken' (the premises) and that the plural 'certain things being supposed' implies that there must be more than one premise. Thus, arguments containing the conclusion as a premise and arguments with only one premise would not be *sullogismoi* (As Alexander tells us, the Stoics included such arguments in their logic). Also note in passing that arguments with no premises and arguments with multiple conclusions (both notions which have been used by some modern logicians) seem to fall outside Aristotle's definition. Nothing in the definition, however, requires that a deduction have exactly two premises nor that it fall into one of the figures Aristotle defines in A 4–6. Aristotle does, in fact, hold that every deduction whatsoever can be transformed into argument in one of the forms of A 4–6, or at any rate a compound of such arguments, but this is for him the result of a lengthy proof, not merely a matter of definition.

On the expression 'certain things having been supposed' (*tethentōn tinōn*) see the 'Note on the Translation' in the Introduction (*tethentōn*, 'having been supposed,' is the aorist passive participle of *tithenai*). Given the mathematical flavor of much of the *Prior Analytics*, it is probably worth observing in passing that the phrase 'a discourse in which, certain things having been supposed' (*logos en hōi tethentōn tinōn*) can be given a different interpretation: *logos* might mean 'relationship,' and the phrase might mean 'a relationship such that when some things are put in it.'

Aristotle rather surprisingly glosses 'resulting through them' with 'needing no further *term*': we might expect him to say 'no further *premise*.'

24b22–26. There has been considerable debate among modern scholars about the notion of a 'perfect' or 'complete' (*teleios*) deduction: good discussions of the problem may be found in Patzig *1968*, 43–87; Corcoran *1973*, *1974b*. I translate this term as 'complete' because Aristotle contrasts such deductions with 'incomplete' (*atelēs*) or 'potential' (*dunatos*) ones and speaks of 'completing' (*perainesthai*, *teleiousthai*, *epiteleisthai*) the latter (but the honorific associations of 'perfect' would not be out of place here). It is clear enough that Aristotle has in mind the difference between a valid argument and an *evidently* valid argument. Alexander stresses the notion that completing is bringing to light what is 'implicit' (*enhuparchei*) or potential in the premises (23.17–24.18, 24.9–11). In modern terms, Corcoran compares the distinction to that between a valid premise-conclusion argument (that is, a set of premises and a conclusion which they imply) and a deduction (that is, an extended discourse which makes it evident that a certain conclusion is implied by certain premises). Note that once again, Aristotle refers to terms where we would expect him to mention premises.

24b26–30. This passage contains what later became known as the *dictum de omni et nullo*, upon which, according to the traditional interpretation, the theory of deduction (and thus all logic) was supposed to be based. Łukasiewicz dismissed this claim as hopelessly confused. But Aristotle himself appeals to this definition as a justification of perfect deductions (e.g., 25b39–40, 26a24–25). There may appear to be a confusion here: if a complete deduction is *evidently* valid, then what need is there for a proof to make its validity evident? And besides, as Aristotle himself argues in *Posterior Analytics* I.3, any system of proofs must rest on some principles not subject to proof, so that it is a mark of ignorance to look for a principle to support every claim (cf. *Metaphysics* IV.3, 1005b2–5). But what Aristotle is doing here is analogous to a modern formal theorist justifying the axioms of a system by offering a model which makes those axioms evidently true. There is nothing inconsistent about his offering justifications for deductions which he characterizes as perfect.

In fact, 25b26–29 is not the only passage which functions in this way in the *Prior Analytics*. Twice in A 14 (32b40–33a5 and 33a24–25), Aristotle justifies his claims that certain combinations of modally qualified premises yield complete deductions by appealing to a definition of possibility (in this case the definition discussed at length in A 13). If indeed these definitions function as semantical principles of meaning and truth, then it is appropriate that Aristotle should have distinct principles defining the truth of simple assertions and assertions of possibility.

24b27. 'every one': the Greek is just 'of every' (*kata pantos*). I am trying to preserve the parallel in Aristotle's language without resorting to the barbarous and unintelligible 'predicated of every of another.'

24b30. 'likewise': not that 'of every' and 'of none' are synonymous, of course, but that 'of none' is defined like 'of every,' *mutatis mutandis* (presumably something like 'none of the subject . . . of which . . . *can* be said').

Chapter 2

25a1. 'Now': the word used here (*epei*) means 'since,' but Aristotle often begins summaries or enumerations this way.

25a3. 'Prefix' (*prosrhēsis*) (which occurs here for the only time in Aristotle's works) refers to the indicators of the three modalities. In other writers, it means 'designation' or 'form of address.'

25a5–13. This section states the conversion properties of nonmodal premises. A proof of these is offered immediately following, in 25a14–26. It is significant that in this opening statement, Aristotle's language is entirely metalinguistic, even at the cost of some awkwardness of expression: that is, he *describes* classes of premises rather than exemplifying them or exhibiting their structures. By contrast, in the proof which follows, he displays premise *forms*, using letters in place of actual terms. As Frede *1974* suggests, Aristotle almost certainly borrows this latter practice from the mathematics of his time: letters are a proof device, not (as Łukasiewicz thought) part of his logical theory itself. Other similarities to mathematical proof are the use of third-person imperatives in 'setting out' the terms (as at 25a14); the recapitulations of results proved after the proof (noted below); and possibly some appeal to diagrams (on which see Einarson *1936*, Rose *1968*).

25a6–8. Although Aristotle's initial definitions distinguish premises into 'affirmative' (*kataphatikos*) and 'negative' (*apophatikos*), he regularly varies each of these terms with a synonym: 'positive' (*katēgorikos*) in place of affirmative, 'privative' (*sterētikos*) in place of 'negative.' These pairs of terms appear to be completely synonymous in the *Prior Analytics*, but Aristotle takes care to differentiate 'negative' and 'privative' in other contexts: the latter term is derived from *sterēsis*, 'privation,' which is one of the four types of opposites enumerated in various places (the opposite of a privation is a 'possession,' *hexis*). Since this terminological peculiarity may be of significance for the study of Aristotle's development, I have regularly translated *katēgorikos* as 'positive' and *sterētikos* as 'privative.'

25a14–26. Aristotle's proofs of the conversion properties rest on the initial proof of the convertibility of *e* sentences. This has, from early times, been challenged as circular since it appears to be an indirect proof with an embedded third-figure deduction (either *Disamis* or *Darapti*), for the proofs of which Aristotle later (A 6) relies in turn on the very conversion properties being proved here. More probably, as Philoponus and Patzig suggest, Aristotle is using the procedure he occasionally calls 'proof through the setting out' (*dia tēs ektheseōs*), which bears a certain resemblance to the existential instantiation of modern predicate logic. Given that *A* belongs to some *B*, we may assign a name (say '*C*') to those *B*s to which *A* belongs; but if *A* belongs to every *C*, then there are some *A*s (viz., *C*) which are *B*s; thus, *B* belongs to some *A*. (Aristotle himself does not mention this procedure until A 6, and he never discusses it in detail. For a survey of interpretations, see Smith *1982b*.) But if this is what he is doing, then it is somewhat odd that he bases his proof of *i*-convertibility on the indirect proof of *e*-convertibility, since, as Alexander notes (33.23ff) a proof of *i*-convertibility by *ekthesis* is almost immediate.

Chapter 3

25a27–36. Aristotle expresses necessity in several ways: he says that a predicate 'belongs of necessity' (*ex anankēs huparchei*) or that 'it is necessary <for a subject> to belong' (*anankē huparchein*), and he says that a premise or conclusion is necessary (*anankaios*). He also uses negated idioms of possibility for necessary negative premises, e.g., 'it is not possible for A to belong to any B.' As the latter type of expression indicates, Aristotle's usage is somewhat flexible, and one probably ought not make too much of the particular idiom used on a particular occasion. However, I have generally tried to associate a single English idiom with each of his Greek constructions.

25a37–b25. In Aristotle's Greek, 'is possible' is expressed with a single verb (*endechesthai*, or less commonly *enchōrein*: I translate these two verbs identically). It is sometimes difficult to reflect this in idiomatic English. Aristotle expresses possibility by saying 'it is possible for A to belong to B' (*to A endechetai tōi B huparchein*); He also describes premises as 'possible' (*endechomenon*) or 'in possibility' (*en tōi endechesthai*). As in other cases, he frequently abbreviates. For instance, we often find the compressed form 'A to B is possible' (*to A endechetai tōi B*), in which 'to belong' is omitted. I have generally filled these out where the meaning is unambiguous.

This discussion of the conversion of possible premises should be compared with the later discussion in A 13. Despite the prospective reference here and a retrospective reference in that later chapter (32b1–3), Aristotle's doctrine in the two passages seems inconsistent or confused. (See the notes on A 13 for attempts at resolving these problems.)

Chapter 4

This Chapter contains the exposition of the first-figure deductions (although we do not hear of the first figure, or indeed of figures at all, until the very last words of the Chapter).

25b26–28. 'Having made these determinations': Aristotle's announced project is quite grand in scope: to determine 'through what premises, when, and how *every deduction* comes about.' As the remainder of Book A makes clear, Aristotle is using 'deduction' in its officially defined sense, in which it applies to an extremely large class of arguments. The subsequent structure of the treatise reflects the project stated here: Aristotle first undertakes to determine all the ways an argument 'in the figures' can come about (Chapters 4–22) and then argues that every deduction without qualification comes about through an argument in one of these figures (Chapter 23). 'Through what premises': compare 43a16–17, 46b38.

25b32–35. This joint statement of *Barbara* and *Celarent* is couched in almost deliberately awkward terminology. The proof, brief as it is, follows in 25b37–26a1.

25b35–37. Aristotle's definitions of 'major,' 'middle,' and 'minor' have caused commentators great difficulty. The problem is that they rely on two incompatible sorts of criteria: both syntactical (the position of the term in Aristotle's standard form for expressing a deduction in the relevant figure) and semantical (the relative extensions of terms). Since the semantical criterion is sensible only for a deduction in *Barbara* with true premises, much ingenuity has been expended on making Aristotle consistent. It is better to suppose that, here as elsewhere, Aristotle is simply less careful than he should be.

25b39–40. Aristotle explicitly appeals to his definition of 'predicated of every/of no' at 24b27–30, as he does again at 26a24 and 26a27: despite the opinions of some commentators, there is no doubt that he takes that earlier passage as a definitional principle of his system.

26a5–9. 'For it is possible': Here, Aristotle uses for the first time his peculiar countermodel technique of term-triple pairs for showing that a certain pattern of premises does not, of itself, give a conclusion. Simply put, he provides two triples of terms concerning each of which premises of the form in question are true, but such that the 'major' term of the first is universally true of the 'minor,' while the 'major' of the second is universally false of its 'minor.' Given Aristotle's understanding of the categoricals, it follows that for any putative form of conclusion, an example can be constructed which has true premises of the appropriate forms and a false conclusion of that form; thus, no deduction results based on the form alone. In this place, Aristotle precedes the technique with a brief explanation. Łukasiewicz (*1957*, 72) objects to Aristotle's introduction of 'concrete terms' into logic and develops an axiomatized rejection procedure to fill what he regards as a gap here (for a similar point, see Geach *1972*, 298–299). Similarly, Ross complains that the use of counterexamples is not 'completely satisfactory' because it introduces extra-logical knowledge. But there is nothing logically flawed in Aristotle's procedure: in fact, countermodels are the paradigmatic means of proving invalidity for modern logicians. The discussion in Patzig *1968*, 168–192, is extremely useful; see also Lear *1980*, 54–61, 70–75, for criticisms of Geach, Ross, and Łukasiewicz.

26a21–23. This definition of 'major' and 'minor' is purely semantical, unlike the partly syntactical definition at 25b35–37.

26a27–28. 'For it has also been defined': here again, Aristotle associates proof by appeal to a definition with the completeness of a deduction.

26b3. The phrase 'whether an indeterminate . . . is taken' (*adihoristou te kai en merei lēphthentos*), which occurs earlier at 26a30, follows here also in most manuscripts. Ross, noting that it makes no sense in this location, omits it as probably a copyist's error.

26b21. Aristotle sometimes uses 'interval' (*diastēma*) as a synonym for 'premise.' The term apparently derives from the Greek mathematical theory of music, as do many of Aristotle's technical terms in the logical treatises. For a discussion, see Einarson *1936*.

26b26–33. The discussion of the first figure ends with a summary of the results proved, as do all the subsequent discussions. Aristotle regularly stresses three points: (1) which deductions in the figure are complete (here, all of them are); (2) how the deductions are completed (here, since all are complete, it is 'through the premises initially taken,' i.e., through *only* those premises); (3) which 'problems' are 'proved' in the figure, i.e., which types of categorical sentences are found as conclusions. In the *Prior Analytics*, 'problem' invariably has this sense (cf. 43a18, 43b34, 45a34–36, 44a37, 45a34–36, 45b21, 46b39, 47b10–13, 50a8, 50b5, 61a34, 63b13–19). In origin, however, it is a technical term of dialectic (like 'premise'). For a discussion of its background, see the Note on 42b27.

Chapter 5

26b34–39. Aristotle's definitions of the second figure and the meanings within it of 'major,' 'minor,' and 'middle' are clearly syntactical in character, with no mention of the relative extensions of terms (the middle is 'outside the extremes and is first in position' in that it is mentioned first in his standard formulation).

27a1–25. Here, for the first time, Aristotle contrasts a complete deduction with a 'potential' (*dunatos*) one. The phrase suggests that an incomplete deduction is only potentially a deduction. Corcoran *1973* argues that this fits well with the view that a 'completed' or 'perfected' incomplete deduction is a valid premise-conclusion argument supplemented by deductive steps which make its validity evident (see also Smiley *1973*). We also find here the first example of the process of completing an incomplete deduction. For a discussion of his procedure, see the Introduction.

27a10. 'neither will N belong to any X': the manuscripts generally have 'neither will X belong to any N'; Alexander also read this in his text and was puzzled by it. In Greek, the two phrases are very close (*to X tōi N* versus *tōi X to N*), so that a mistake here is easy (as anyone who has attentively read long stretches of this type of prose can affirm). Since Aristotle's practice is almost always to state in advance the conclusion he is about to deduce, I have switched the variables. (But see also Patzig *1968*, 140 n.18.)

27a14–15. Aristotle mentions here for the first time the procedure of 'leading to an impossibility' (*agein/apagein eis to adunaton*) as an alternate way of completing an incomplete deduction (for a discussion of his procedure here, see the Introduction). The technique is introduced and used without explanation or discussion, most probably because Aristotle expected it to be familiar to his audience from its use in Greek mathematics and in philosophical arguments (notably by the Eleatics). There is a problem about the relationship of his *use* of proof through impossibility here to his *discussions* of the technique in A 23 (41a21–37), A 29, and B 11–14. In those later passages, he claims that whatever can be deduced or proved through impossibility can also be deduced or proved 'probatively' (*deiktikōs*), i.e., in modern terms, directly (see especially A 29, B 14, and the associated Notes). This would imply that proof through impossibility is simply a redundant technique, and in these places that appears to be just what Aristotle is urging. But although completions through impossibility are sometimes mentioned as mere alternatives (as here), in other cases they are essential (as in the completion of second-figure *Baroco*, 27a36–b3, or third-figure *Bocardo*, 28b17–20). In A 45, in fact, Aristotle's intent appears to be to establish that proof through impossibility is the *only* means for completing these very cases.

We might try to reconcile these passages with the suggestion that A 23, A 29, and B 11–14 are all concerned with the analysis of actual arguments, using the theory of deductions in the figures as a means for that analysis, whereas in

A 4–22 we are establishing that theory itself. A modern logician might be tempted to see here a distinction between deductions within the system and results proved about the system (in modern terms, *metatheoretical* results). Aristotle could then be seen as holding that, within the system itself and its applications, proof through impossibility is redundant, even though it is essential for the establishment of the system. But however much kinship one may see between Aristotle and his twentieth-century successors in logical theory, it seems to me very difficult to find grounds for imputing such a distinction to him.

27a16–18. 'not from the initial premises alone, but from others': all Aristotle actually says is 'from the initial <things>' (*ek tōn ex archēs*) and 'from others' (*ex allōn*), but he always describes a deduction as 'from' (*ek*) its premises. The 'other premises' are the intermediate steps in the deduction, which Aristotle regards as distinct premises (cf. 28a4–7, and the associated Notes). The 'necessary result' (*to anankaion*: literally, 'the necessary thing' or 'the necessity') is the conclusion: since Aristotle regards the conclusion as necessary because it necessarily results from the premises, he often uses this designation for it. And, since there is a deduction only if there is a conclusion, Aristotle frequently speaks as if the properties of the conclusion of a deduction are properties of the deduction, or conversely, and takes the conclusion as representing or summing up the deduction itself. Thus, here it is the *conclusion* that is 'completed.'

27a36–b3. In this proof of *Baroco* Aristotle gives us an explicit completion through impossibility for the first time (though he does not identify it as such). It is noteworthy that he runs through the proof twice: once with the *o* premise stated in the form 'M does not belong to some X,' and once with it in the form 'M does not belong to every X.' (He does not, however, undertake to show that both forms of *o conclusion* follow when there is no *o premise*, as in the case of *Festino*.) Compare B 11, 62a9–10.

27b20–23. Here Aristotle introduces a sophisticated modification of his countermodel technique, which he calls 'proof from the indeterminate.' The difficulty is that Aristotle usually treats the particular categoricals as strictly particular: 'A belongs to some B' means 'A belongs to some *but not every* B.' But if we apply such an interpretation to the particular affirmative premise here, we in effect have the premises of *Festino*: as a result, it is not possible to find 'terms for belonging' (that is, terms satisfying the premises and with the major universally true of the minor) under this interpretation. Aristotle's response is that: (1) the truth of 'M belongs to no X' is sufficient for the truth of 'M does not belong to some X'; (2) it has already been shown that 'M belongs to no N and to no X' gives no conclusion. He thus invokes the principle that if a set of premises yields no conclusion, then the set that results from it when one of the premises is replaced by a weaker premise also yields no conclusion.

Aristotle's designation of this procedure as 'from the indeterminate' indicates what the fundamental meaning of 'indeterminate' is. Any given particular sentence is made true both by the circumstances under which it is *strictly* true ('some but not every') and by the circumstances under which its corresponding universal sentence is true. Consequently, if we know only that a certain particular sentence is true, then we do not in virtue of that knowledge know which of these circumstances obtains. What is 'indeterminate' about particular sentences in these cases is that the truth of the sentence does not imply that a *unique* state of affairs (as Aristotle sees it) holds. We find a similar use of 'indefinite' (*ahoristos*) in the case of possible premises: see the Notes on 32b4–22.

27b20. 'must be proved' (*deikteon*): The verb *deiknunai* can have the weak sense 'show' (or even 'point out') as well as the strong sense 'prove.' However, in the *Prior Analytics*, Aristotle commonly uses it interchangeably with 'demonstrate' (*apodeiknusthai*). The stronger sense is quite appropriate here: showing that certain types of premises do not yield a conclusion is just as much a matter of proof as showing that others do. (But see the Note on 38b21–22.)

27b21. 'There was not a deduction' (*ouk ēn sullogismos*): i.e., '*as we have seen*, there was not a deduction.' The use of the past (imperfect) tense to indicate a result previously established is extremely common in the *Prior Analytics*.

28a4–7. In explaining why the second-figure moods are incomplete, Aristotle gives us a better idea what it is that must be 'taken in addition' to complete an argument: either intermediate steps which are 'implicit in' the premises or assumptions for proofs through impossibility (*reductio* hypotheses). In either case, it is evidently the appearance of a statement in the completed deduction which counts as 'taking' it. See the notes below on A 23, A 29, B 14 concerning difficulties in Aristotle's understanding of argument through impossibility.

The word I translate 'are implicit in' is *enhuparchei* (literally 'belong in,' 'be present in,' 'exist in'). This word has a number of uses in Aristotle and sometimes has a technical force associated with the constituents of a definition. However, it also is used in the biological works of the incomplete parts of immature animals and in other contexts where it clearly suggests something present but not yet discernible, developed, or brought out.

Chapter 6

28a10–15. These definitions of the third figure and the meanings in it of 'major,' 'minor,' and 'middle' are essentially syntactical, as in the case of the second figure.

28a22–26. Aristotle again alludes to the possibility of a proof through impossibility, though he does not give the details, and introduces his third proof technique: 'through the setting-out' (*tōi ekthesthai*). For a discussion of this procedure see the Introduction.

28a28. 'of necessity does not belong': the position of the phrase 'of necessity' in this sentence appears to indicate, not just that the conclusion follows of necessity, but that it is itself a necessary conclusion (in the discussion of modally qualified deductions in A 8–22, exactly the same sort of phrase would probably have just that meaning). Aristotle does not mean that, of course (see the Notes below on 31a6–7, 36a17–18).

28b17–21. Aristotle explicitly gives a proof through impossibility for *Bocardo* and then notes that this can be avoided by a procedure evidently identical with the proof 'through the setting-out' used for *Darapti* (28a22–26). The present passage suggests that Aristotle may have regarded proof through *ekthesis* as a (preferable?) alternative to proof through impossibility. The two procedures are often mentioned together; later, in A 9, Aristotle appeals to *ekthesis* expressly because an attempted proof through impossibility fails. (In Smith *1983* I show that a formal version of Aristotle's deductive theory lacking proof through impossibility, but containing *ekthesis*, is still complete. Although there is no direct evidence that Aristotle realized this, it is at least conceivable that he saw *ekthesis* as a way of avoiding proof through impossibility.)

Note his characterization of the proof as 'a leading away' (*apagōgē*). This term, which might be Latinized as 'abduction' or 'deduction,' is usually associated in his usage with proof through impossibility, but it is defined in a more general way in B 25 as the substitution of one 'problem' for another (see the Notes on that chapter).

Chapter 7

29a19–29. Aristotle here adds a note that effectively brings the fourth-figure moods *Fesapo*, *Fresison* into his system. His basic point is a simple one: *e* premises are convertible into *e* premises, while any affirmative premise can be converted to yield an *i* premise (which is again convertible), and, therefore, any combination of an *e* premise and an affirmative premise can be worked around by means of conversions into the form of *Ferio*. In all those cases in which no deduction resulted in the three figures, this is because the resulting conversions have the effect of reversing major and minor terms; Aristotle takes note of this fact here.

29a30–39. The result announced here, together with the claim which follows, are among the most important conclusions Aristotle draws from his study of deductions. Briefly, his claim is that if a deduction in one of the figures is possible at all, then it is possible through a first-figure deduction. He appears to take this to mean that every deduction can be *transformed into* a first-figure deduction. This passage contrasts completing a deduction 'probatively' (*deiktikōs*) and completing it through an impossibility. The natural, modern equivalent of *deiktikōs* here is 'directly,' but I have chosen 'probatively' in order to preserve the connection with *deiknunai*, 'prove' (and, in any event, Aristotle has no equivalent for 'indirect'). See further the discussion of A 23, 40b25. The word translated 'come to a conclusion' (*perainesthai*) could also be translated 'come to a goal' or 'be completed,' and thus it could have the same meaning as 'complete' (*teleiousthai*).

29b1–25. Aristotle closes by proving an important result about his system (in modern terminology, a metatheorem): every deduction in the three figures can be completed by means of the two universal deductions of the first figure. The argument is economically organized: (1) this already holds for the second-figure deductions; (2) the first-figure particular deductions can be completed through second-figure universal deductions (which are, in turn, completed by the two universal first-figure deductions); (3) the third-figure deductions are all completed through first-figure deductions (and the particular deductions of the first figure have just been shown to be completable through the universal first-figure deductions). This is a sophisticated result which, in its elegance of presentation, is evidence of Aristotle's level of technical expertise. I argue in Smith *1986* that this result is of crucial importance in one of the central arguments in the *Posterior Analytics*.

Here, for the first time, Aristotle refers to 'leading back' (*anagein*) one deduction into another. The process is indistinguishable from the proofs of Chapters 4–6, but the new verb reinforces the suggestion that the process is one of analysis into elements or principles. Possibly, this passage was added some time after Aristotle originally composed Chapters 1–6, as Łukasiewicz and Bocheński proposed. The traditional translation of *anagein* is 'reduce,' which masks the real sense of the word. On this procedure, see the Notes on 46b40–47a2.

29b26–28. The deductions which 'prove something to belong or not to belong' are those which contain no modal terms (in other words, everything considered so far). The point of the phrase is to contrast these results with those about to be established in the next part of the treatise.

Chapter 8

Chapter 8 begins the exposition of deductions with modally qualified premises. Aristotle first discusses combinations of two necessary premises (8) and of one necessary and one assertoric premise (9–11) in all the figures. He then discusses combinations involving possible premises, treating each figure separately: first two possible premises, then one possible and one assertoric premise, then one possible and one necessary premise. The first figure is studied in 14–16, the second in 17–19, and the third in 20–22. Interspersed in the treatments of individual deductions and inconcludent premise pairs, there are a brief summary note (12), a second and fuller discussion of the conversion of possible premises (13), a lengthy and difficult discussion related to the use of proof through impossibility in modal cases (15, 34a5–18), and an argument that possible *e* premises do not convert analogously to their assertoric and necessary counterparts (17, 36b35–37a26).

For discussion of Aristotle's idioms for expressing the modalities, see the Notes on A 3. He distinguishes assertoric premises (premises without a modal qualifier) as assertions 'of belonging' (*tou huparchein*), and regularly uses 'belonging' (*huparchon*) in parallel with 'necessary' (*anankaios*) and 'possible' (*en-*

dechomenos) to describe the modal status of premises. The verb *huparchein* can mean 'be the case' as well as 'belong,' and therefore there is a certain ambiguity in all these expressions: we might translate 'since to be the case, to be the case of necessity, and to be possible to be the case are all different . . .' However, the use of *huparchein* in connection with the relation of predicate to subject is enormously frequent in the *Prior Analytics*.

29b33–35. This summary rather crudely implies that the conclusion will have the same modality as the premises: the view Aristotle actually espouses is more complex, since he thinks that in some cases an assertoric conclusion can be deduced from a necessary and a possible premise. This may be the result of carelessness or excessive brevity, but Aristotle misdescribes the results of his study of modal deductions elsewhere (see in particular A 12): an alternative explanation, which I would favor, is that these remarks were written before the details of A 8–22 had been worked out.

29b36–30a3. Aristotle asserts that a deduction with necessary premises and a necessary conclusion is possible if and only if a parallel deduction with assertoric premises and conclusion exists. In general, his account of modally qualified deductions rests on the theory of assertoric deductions: he only investigates those premise combinations which he already knows to yield a conclusion in their assertoric forms (this does not quite hold for possible premises because of 'complementary conversions').

30a2–3. Aristotle appears to claim here that two points suffice to establish the parallel between pairs of assertoric and pairs of necessary premises: (1) (universal) negative premises convert in the same way, whether assertoric or necessary, and (2) the definitions of 'in a whole' and 'of every' from 24b26–30 carry over to this case. Presumably, what he has in mind is that all the proofs offered in 4–6 can be duplicated in the case of necessary premises. But two points are puzzling. First, why does he only mention negative premises, since the conversions of affirmatives are also essential to those earlier proofs? Ross supposes him to have in mind the fact that, whereas necessary *e* premises convert analogously to their assertoric counterparts, possible *e* premises (according to A 17) do not (and he suggests that Aristotle simply does not bother to mention the convertibility of affirmatives). This may be the best answer, but it does take Aristotle to be speaking in an extraordinarily elliptical manner. The second puzzle, and a more serious one, is the fact that Aristotle proceeds at once to give a case for which these arguments are insufficient.

30a3–14. *Baroco* and *Bocardo* are the only deductions for which Aristotle cannot give proofs by conversions; thus, the argument of 29b36–30a3 will not do for their case. As A 45 shows, Aristotle realizes that completion through some means other than conversions is unavoidable for the assertoric deductions in these cases. In A 5 and A 6, he gave proofs through impossibility. However, in the present case he evidently believes that he cannot use such an approach. Accordingly, he resorts to proofs by 'setting-out,' parallel to those given earlier for *Darapti* and *Bocardo*. This again reinforces the notion that such proofs were conceived by him as an alternative to proof through impossibility.

The customary explanation why Aristotle cannot use proof through impossibility is that it requires him to appeal to a deduction with mixed premises (since the denial of a necessary premise is not a necessary premise). For instance, in the proof given in A 5 for *Baroco* (27a36–b1), Aristotle uses the denial of the conclusion and the major premise to deduce the denial of the minor premise. In the present case, this would require a deduction in *Barbara* with possible major and necessary minor, yielding a possible conclusion. Ross says that Aristotle cannot use this because he 'has not yet examined the conditions of validity in mixed syllogisms.' But when Aristotle does get around to this case (A 16, 36a2–7), he tells us that it is a complete deduction: why then does he not appeal to that fact here?

30a12–13. 'is just a certain "that"': this phrase is difficult to put into graceful English, though the sense is clear enough. Aristotle means that if the predicate belongs (or does not belong) of necessity to the 'term set out,' then it likewise belongs (or does not) of necessity to *some* of the original term of which the 'thing set out' was taken as a part. The complication is Aristotle's reliance on the Greek convention of using 'that' (*ekeino*) as we use 'the former': the phrase *ei de kata tou ektethentos estin anankaios, kai kat' ekeinou tinos: to gar ektethen hoper ekeino ti estin* might be rendered 'if it is necessary of what is set out, then it will be necessary of some of the former <sc., the original term in the deduction>: for the thing set out is just a certain "former."' Some commentators take *hoper ekeino ti* here as indicating that the term set out is a sensible particular, rather than a universal, and conclude from this that the procedure of 'setting out' relies in some way on sensory perception. But Aristotle most frequently uses *hoper* as a technical locution to indicate the essence or nature of something: saying that X is *hoper* Y is equivalent to saying that X is essentially a Y. Thus, he need only be saying that 'the thing set out is essentially a certain so-and-so.'

30a14–15. Each proof through setting-out relies on a deduction in the same figure (second or third, respectively) as the original deduction.

Chapter 9

30a15–23. The best-known difficulty with Aristotle's account of modal deductions is that he holds that it is sometimes possible to deduce a necessary conclusion from premises not all of which are necessary. Many proposals have been offered for devising an account of necessity which will accommodate this position or, failing such a defense, for explaining why Aristotle is led to take it: see Łukasiewicz *1957*, 181–208; McCall *1963*; Rescher *1964*; Patzig *1968*, 67–69. But Hintikka (*1973*, 135–146) persuasively argues that all attempts at a formal model are doomed to failure by inconsistencies in Aristotle's basic views about modalities. Note that Aristotle's proof procedure again relies on *ekthesis*, as indicated by the reference to 'some of the Bs' in 30a22.

30a25–28. Aristotle gives a counter-argument for the case of a necessary minor premise: if we accept the validity of such a deduction, we can derive something which clearly does not follow from the premises (in the case given, we start with premises 'A belongs to every B' and 'B belongs of necessity to every C' and deduce 'A belongs of necessity to some B'). Aristotle's technique is sophisticated and flawless: he notes that the entire *inference* from the original premises to 'A belongs of necessity to some B' is invalid by giving (in the abstract) a counterexample, and then he concludes that the inference *rule* which gave rise to this must, therefore, be invalid.

30a27. 'this is incorrect': Aristotle actually says *pseudos* ('false'), but here, as occasionally elsewhere, he applies it to a rejected rule of inference (see 37a2, 37a22, 48a16, 49a18–22, 52b20, 52b28).

30a35. Although Aristotle actually says 'the conclusion will not be necessary,' it does not actually *follow* in this case (and similar cases) that the conclusion is not necessary but instead only *fails to follow* that it must be. We may take Aristotle to mean that the inference in this case is invalid.

30a40. 'C is under (*hupo*) B': 'under' in this sort of context usually means either 'within the extension of' or 'a subject of predication of.' In the present case, it has to mean something like 'part of C falls under B,' if Aristotle's argument is to work. He must mean his proof to follow the same form as that in 30a21–23, though his expression is perhaps careless.

30b4. 'Nothing impossible results': that is, nothing impossible *would* result from supposing the conclusion not to be necessary. The reference to the case of 'universal deductions' is to 30a28–33, and what Aristotle gives us there is a proof through terms that a non-necessary conclusion is consistent with premises of the relevant types. This should be seen as the complement to the procedure of deducing an impossibility: we show that a set of premises *is* possible by producing a conceivable case in which all its members are true, and we show that a set of premises is *not* possible by deducing a contradiction from it.

Chapter 10

30b10. Note that Aristotle readily expresses a negative necessary premise as a denial of possibility. Later this gives rise to ambiguities: see the Notes on A 14, 33a3–5; A 16, 36a17–18; A 17, 36b35–37a32.

30b24. 'Of necessity' is ambiguous: it could mean either that the conclusion which follows *cannot* be 'of necessity' (i.e., necessary), or that it is not of necessity (*need* not be) a necessary conclusion.

30b33. 'Necessary when these things are so' (*toutōn ontōn anankaion*) is Aristotle's usual way of expressing 'hypothetical necessity.' His doctrine on this point is alien to modern logicians: Aristotle takes the sentence 'If *A*, then necessarily *B*' as attributing a kind of necessity ('hypothetical' necessity) to *B*. The point which he makes in the present case actually applies to any assertoric deduction, since (according to his definition) the conclusion of a deduction follows of necessity from the premises: thus, the conclusion of any deduction is necessary-if-the-premises-are-true (cf. the Note on 27a16–18).

31a6–7. 'let it not be possible for A to belong to any B': word for word, 'let A to no B be possible' (*to A tōi B mēdeni endechesthō*). Aristotle often expresses a necessary *e* premise this way (the general form is 'to none is it possible for A to B to belong': *oudeni endechetai to A tōi B huparchein*). An almost identical idiom serves for possible *e* premises: 'it is possible for A to no B to belong' (*endechetai to A oudeni tōi B huparchein*). The crucial distinction is the order of 'is possible' (*endechetai*) and 'to no' (*oudeni/mēdeni*): when *oudeni* precedes *endechetai* the meaning is 'necessarily not to any,' while otherwise the meaning is 'possibly not to any.' (See the Note on 36a17–18.)

Chapter 11

31a31–32. 'C converts to some A' (*antistrephei gar to G tōi A tini*): this very elliptical phrase must mean 'by converting the premise AC, we have the premise that C belongs to some A.'

31b8–9. 'Not possible' here is *mē dunaton*, not *ouk endechomenon*. In Aristotle, *dunatos* usually has the stronger sense 'potential,' but in the *Prior Analytics*, we occasionally find it as a synonym for *endechomenos*. Here, it is not actually applied to a premise but to a situation. See the Notes on A 13 and on A 15, 34a5–12, for further remarks.

31b15–16. 'the previous one': at 31a24–37.

31b28. 'wakefulness': this is not an attributive term but (in modern terminology) an abstract singular term; as Aristotle himself notes later in A 34, we do not predicate 'wakefulness' of something, but rather 'awake' (as indeed we find a few lines later in the example). What this indicates, I think, is that Aristotle regards premises and deductions, not as essentially linguistic entities composed of certain words, but as relationships of nonlinguistic terms. On such an understanding, to say that X is awake does indeed predicate wakefulness of X, though it does not predicate 'wakefulness' of X (that would be done by a sentence like 'X is wakefulness'). In the background of this is a theory of the subjects of predication as 'paronymous' with—'named after'—the qualities or properties attributed to them. Thus, things which possess wakefulness are called, not wakefulness, but 'awake,' and things which possess equality are called, not equality, but 'equal.' Aristotle only gives us sketches of this picture in the treatises (*Categories* 8, 10a28–b7; *Topics* II.2, 109a39–b12; *Eudemian Ethics* III.1, 1228a36); it is reminiscent of Plato (compare, e.g., *Parmenides* 130e5–131a2, where things which partake of Forms are said to be 'called after the name' of the Form). See the Notes on A 34 for a fuller discussion.

31b29–30. Aristotle actually expresses the second premise and the conclusion very elliptically here: 'the A to C is possible and the A to B is not necessary' (*to de A tōi G endechetai, kai to A tōi B ouk anankaion*). They can be filled out from the specification of the case at 31b20–23.

31b39–40. 'the proofs': all Aristotle says is 'the rest' but the phrase is parallel to others in which he adds 'proof' (*deixis*) or 'demonstration' (*apodeixis*).

Chapter 12

The results summarized in A 12 complement those of A 24. Aristotle's point is more difficult to state clearly than at first appears; since he has already argued that a number of deductions with one necessary and one assertoric premise have assertoric conclusions, his first claim seems to be false. Commentators usually take him to presuppose an ordering of modalities in terms of strength, with necessity the strongest and possibility the weakest. He would then be asserting here that a necessary conclusion may follow from premises not all of which are *at least* necessary, whereas an assertoric conclusion only follows from premises all of which are *at least* assertoric. But all Aristotle has considered up to this point is necessary and assertoric premises, and to apply this interpretation to his present assertion is strained. Moreover, it is flatly inconsistent with his later claims (A 16, 36a7–17, 34–39; A 19, 38a16–26, 38b8–13, 25–27; A 22, 40a25–32, 40a40–b8) that some combinations of a necessary and a possible premise yield an assertoric conclusion. A more likely view, in my opinion, is that the passage is simply a defense of the claim that a necessary conclusion can follow from one necessary and one assertoric premise. This was challenged by his associate Theophrastus, who argued instead for the rule that in application to modalities, the conclusion always has the weakest modality exhibited in the premises. A 12 may be an attempt to draw a parallel with the facts about quantity and quality summarized in A 24. Every deduction must have one affirmative premise and one (not necessarily distinct) universal premise; a deduction has a negative conclusion if and only if it has a negative premise; and a deduction with a particular premise has a particular conclusion. These rules might be formulated as remarks about 'the other' premise, e.g., a deduction must have an affirmative premise, and *the other* premise must be like the conclusion (in quality). The parallel cannot be made exact, however. Once again (cf. A 8, 29b33–35), either Aristotle is careless or this passage antedates the full study of modal deductions.

It is worth noting that Aristotle refers to this same thesis in the *Rhetoric* (I.2, 1357a27–30) and cites the *Analytics*. If the reference is indeed Aristotle's and not the work of a later reconciling editor, this suggests that the *Rhetoric* antedates the final form of Book A.

Chapter 13

32a18–21. As given in this passage, Aristotle's official definition of the senses of 'to be possible' (*endechesthai*) and 'possible' (*to endechomenon*) is 'not necessary but not entailing anything impossible.' The remark that 'we call the necessary possible only equivocally' is an acknowledgment that there is a sense of 'possible' (to wit, 'not impossible') which applies to what is necessary but that Aristotle is not defining that sense here. (Subsequently, Aristotle often refers to possibility in this sense as possibility 'not according to our definition.') Aristotle's definition seems to be logically equivalent to 'neither necessary nor impossible.' Waterlow *1982* 16–17 explains its prolix form by proposing that it offers a test for possibility rather than a definition of it; she argues further that the test is linked to the notion of 'relative temporalized' possibility which she claims is fundamental for Aristotle.

32a21–29. These lines, which offer a defense of the preceding definition of 'possible,' are extremely difficult to reconcile with the surrounding text. The style of argument, reminiscent of *On Interpretation* 12, revolves around the determination of the contradictories of expressions (the underlying principle is that the contradictories of equivalent expressions are equivalent). What the argument begins with is plausible enough, but on the definition of 'possible' just given it leads to an absurdity. If what is necessary is not possible, then 'is not possible to belong' follows from 'is necessary to belong.' But if we take Aristotle to mean here that the three expressions he gives entail one another, then 'is necessary not to belong' follows, in turn, from 'is not possible to belong'; therefore, we can derive 'is necessary not to belong,' from its contrary 'is necessary to belong.' Hintikka *1973* tries to resolve this by interpreting 'follow' (*akolouthein*) as expressing consistency rather than entailment; but even if we accept this (which I think we cannot), we still get the absurd result that 'is necessary to belong' is consistent with its contrary.

The inference from the equivalence of the three expressions to the equivalence of their contradictories is unproblematic, but it leads only to the conclusion that 'possible' is equivaent to 'not impossible' and 'not necessarily not.' The final conclusion that 'possible' is equivalent to 'not necessary' does not follow from this, nor is it clear how to reconcile it with Aristotle's officially announced definition of possibility. Taken literally, it has the absurd consequence that whatever is impossible is possible (since the impossible is not necessary). However, Aristotle probably means to include the impossible as a species of the 'necessary' in the sense that what is impossible is necessarily *not* the case; so interpreted, the conclusion is a statement of Aristotle's standard doctrine, though its relationship to the premises from which it is supposed to follow is then problematic.

A number of scholars, beginning with Becker *1933*, have argued that these lines are an inept interpolation designed to reconcile Aristotle's definition of possibility here with the wider definition of *On Interpretation* 12. Hintikka, however, defends their authenticity by suggesting that the passage is highly elliptical and 'concise' (*1973* 31–34). On his view, Aristotle is arguing for the definition of 'possible' as 'neither necessary nor impossible' by temporarily discussing some of the consequences of the *alternative* view ('not impossible') in order to show that they are unacceptable. But while this would be an attractive interpretation, it requires us to suppose some very substantive (and in my opinion implausible) ellipses: it is more likely that an interpolator has been at work.

32a29–b1. Here, Aristotle introduces what Ross calls 'complementary conversion.' As is argued in 32a36–b1, if 'possible' means 'neither necessary nor impossible,' then 'possible to belong' entails 'not necessary to belong,' which, in turn, entails 'possible not to belong.' As a result, we may add or remove 'not' within the scope of 'possible' with preservation of equivalence: 'it is possible for A to belong to B' entails and is entailed by 'it is possible for A not to belong to B.' Aristotle extends this to all the quantified forms of predication. On those which 'have an affirmative form,' cf. the immediately following sentence.

32b1–3. 'as was stated earlier': the reference is to A 3, 25b19–25, where Aristotle asserts that 'is possible,' like 'is,' 'always and in all ways makes an affirmation' of that to which it is added. In that earlier passage, Aristotle said that this claim would be 'proved through what follows': evidently, the present passage is the promised proof.

32b4–22. Aristotle here distinguishes two cases of 'possible' understood as he has defined it: what generally or naturally happens, and what may equally well happen or not happen. His intent is to defend complementary conversion for both cases. Such a defense would seem to be otiose in view of the simple general argument of 32a36–b1, but Aristotle's method yields an insight into his understanding of logic: he regards the present argument as giving the *reason why* possible premises of each of the two sorts admit complementary conversion. This is probably related to his general position that 'verbal' (*logikos*) arguments cannot actually explain *why* something is so, even though they may establish *that* it is.

The Greek commentators take Aristotle's distinction to have an important statistical component: what is 'for the most part' is what happens more often than not in a given case, whereas what is 'indefinite' is what happens or fails to happen with equal frequency (both Alexander and Philoponus suppose that this category also includes what generally does *not* happen, the complement of what generally happens). But there is evidence, both in the present passage and elsewhere in Aristotle's works, that the distinction rests on very different grounds.

'For the most part' (*hōs epi to polu*) in Aristotle means 'what ordinarily happens in the usual course of events'; his designation of it as 'natural to belong' (*pephukos huparchein*) reflects the fact that for him such things have their origins in the natures of things. The 'indeterminate' (*ahoristos*) type of possibility actually embraces two very different cases. Aristotle first refers to what is 'capable' (*dunaton*) of being thus or not thus. This term reflects a view of capacities as occurrent properties of things (that is, potentialities) and is closely bound up with Aristotle's doctrines concerning potentiality and actuality. He often tells us that capacities of this sort are intrinsically two-sided:

the capacity to walk, for instance, is identically the capacity not to walk. (In some places, e.g., *Metaphysics* IX.5, he distinguishes 'rational' capacities, which are two-sided in this way, from 'irrational' ones, which are not.) His second case, however, is quite distinct: a coincidence of unrelated events. This is often the meaning of 'by chance' (*apo tuchēs*). A fundamental distinction between these two last cases is that whereas the existence of a capacity provides an explanation for its exercise (as an animal's capacity to walk is part of the explanation why it is now walking), coincidences which happen 'by chance' simply have no explanation.

Another important difference could be described as a matter of the semantic basis of the modality. Aristotle tends to regard possibilities strictly so called as matters of the inherence of properties in subjects: 'This man is possibly grey' asserts, of this man, that he possesses the potentiality or capacity of being grey. However, such an analysis does not fit well with logically complex propositions such as 'It is possible that it will thunder while I am walking' or 'It is possible that when I go to the well to get a drink as a result of eating spicy food, I will be killed by passing brigands' (cf. Freeland *1986*). We might put the point by saying that Aristotle tends to think of modalities only in *de re* terms; the *de dicto* analysis required for these latter examples is generally suppressed in his considerations. (For Aristotle's views on chance, see *Metaphysics* VI.3.)

The term 'indefinite' (*ahoristos*) is close in meaning to 'indeterminate' (*adihoristos*). As noted above (27b20–23), Aristotle regards particular premises as 'indeterminate' in the sense that the conditions under which they are true are complex. Similarly, 'indefinite' possibilities here are defined disjunctively: *either* what is potential *or* what is coincidental. In general, faced with a type of sentence which is true under several distinct types of circumstances, Aristotle tends to regard one set of circumstances as the primary truth condition, and others as secondary but not ruled out. Thus, he sometimes says that a deduction leads to a possible conclusion when he means that it leads to a non-impossible one, although it does not lead to one possible in accordance with his official definition. A modern logician (and many a commentator on the *Prior Analytics*) would define two distinct technical terms for these two senses of 'possible' instead of retaining a single one and noting that it sometimes means one thing and sometimes means another.

32b18–22. In the *Posterior Analytics*, the term 'science' (*epistēmē*) is applied to the epistemic state resulting from demonstration or proof (*apodeixis*). On Aristotle's conception, there can be science in this strict sense only of what is necessarily true, not of 'what can be otherwise.' The present point indicates how limited the range of Aristotelian science is when so understood: what happens this way or that without any natural proclivity one way or the other would include a great many of the particular facts about the world. The middle term is 'disorderly' (*ataktos*) in that it fails to have any determinate (and hence knowable) relationship with the extremes. The term *ataktos* has military associations, as do a number of Aristotle's technical terms concerning proof. Compare the use of *tattesthai*, 'be arranged,' at 32b3 above, and the much-discussed military metaphor of the 'rout' in *Posterior Analytics* II.19, 100a12ff.

32b23. 'These things will be better determined': it does not appear that Aristotle ever does so anywhere in his surviving works.

32b24–37. In this passage, Aristotle makes another distinction of sense of possible premises. The ambiguity he intends to call attention to only arises when both terms of the premise are general terms, and even then it is more believable as an ambiguity of Aristotle's preferred locution *kath' hou to B, to A endechetai*—word for word, in roughly equivalent English, 'it is possible that A of what B is of.' Aristotle seems to be concerned by the similarity of this construction to such locutions as 'A is predicated of what B is' (*kath' hou to B, to A katēgoreitai*), which he often uses to express categorical sentences. Here, he rather surprisingly opts for interpreting the sentence in question as 'it is possible that A <is predicated> of what *it is possible* that B <is predicated> of.' One consideration may be the need to have a single middle term in a deduction. If we regard 'it is possible that A to B' as attributing the predicate 'possibly A' to B, and likewise 'it is possible that B to C' as attributing the predicate 'possibly B' to C, then these two premises appear to contain four terms: 'possibly A,' 'B,' 'possibly B,' and 'C.' On the interpretation which Aristotle advocates, there are only three terms, but they are 'possibly A,' 'possibly B,' and 'possibly C.'

Aristotle's reference to 'those the same in form' (*homoioschēmones*) is puzzling. The Greek commentators take him to mean that the interpretation of 'A possibly belongs to B' as 'A possibly belongs to what B actually belongs' leads to a deduction with one possible and one assertoric premise. But there is only one premise here. What Aristotle may be thinking of is something akin to the procedure of *ekthesis*.

32b25–31. 'Now . . . Therefore, it is evident that': The 'therefore' (*epei*) actually occurs at the beginning of this long sentence, in 32b25. I move it, for clarity, to the beginning of the clause it explains, beginning at 32b31.

32b32–33. We could read the text here as saying either 'it is possible that B of what it is *possible* that C' or 'it is possible that B of what it *is* <sc. true> that C.' Owen and Jenkinson explicitly opt for the latter, Tredennick for the former. In view of the preceding discussion, Tredennick probably is right, but I have tried to preserve the ambiguity.

Chapter 14

32b39. 'There will be a complete deduction': as in the case of assertoric first-figure deductions (25b39–40, 26a24), Aristotle defends this claim by asserting that it follows from a definition (here, the definition just offered in 32b25–37).

33a3–5. Aristotle similarly appeals to the definition of possibility to establish the negative case. Here, he gives us a fuller form of the definition he is relying on, which combines the definition at 32b35–37 with the definition of 'belongs to every/to no' at 24b28–30: note the reference to 'not leaving out' (*mēden apoleipein*) anything which is under the subject term. The context clearly requires that the awkward expression 'for it to be possible that not A of that of which it is possible that B' (*kath' hou to B endechetai, to A mē endechesthai*) must mean 'for everything to which it is possible that B belongs, it is possible that A does not belong to it.' However, this is made particularly difficult by the phrase *to A mē endechesthai*: 'A not to be possible' or 'it not to be possible that A.' The 'not' (*mē*) would naturally be taken to go with the infinitive 'to be possible,' giving the phrase the sense 'for it *not to be possible* that A,' which is clearly not what Aristotle intends here. This may explain why some manuscripts omit 'not,' even though this makes Aristotle suddenly revert to discussion of the first deduction (32b38–40). But it is quite in accordance with Aristotle's way of abbreviating premises to let 'A not' mean 'A not to belong' (*to A mē huparchein*). I translate with a modern-sounding 'not A.'

33a12–13. 'negation' (*apophasis*) here clearly means 'sign of negation' (i.e., the word 'not').

33a17–20. Aristotle's remark here implies that at least some combinations of universal premises in the first figure do not yield a conclusion. But he has adopted it as a rule that a possible universal affirmative and its corresponding universal negative are, in effect, equivalent: it follows that *any* combination of universal premises in the first figure entails *both* a universal affirmative *and* a universal negative conclusion. Aristotle only lists four of these eight possible cases: *aaa*, *eae*, *aee*, *eee*. The reason may be that the other four (*aae*, *eaa*, *aea*, *eea*) all contradict the rules he later states (A 24) that an affirmative conclusion follows from two affirmative premises and a negative conclusion from an affirmative and a negative premise. (But that should not disturb him, since he has already maintained that all possible premises are really affirmative in form.) Similarly, in discussing first-figure deductions with particular premises, he should again include all possible combinations (*aii*, *eii*, *aoi*, *eoi*, *aio*, *eio*, *aoo*, *eoo*): he mentions only *aii*, *eio*, *aoo* (and perhaps implies *eoo*).

33a21–22. The text as it stands here claims that *any* combination of a possible universal major premise and a possible particular minor in the first figure yields a complete deduction; but as Waitz and Ross point out, 33a27–34 says that *aoi* is an *incomplete* deduction. These commentators accordingly doubt the authenticity of 'complete' here. But Aristotle justifies the deduction *aii* (and *eao*) by appeal to a definition, which he normally does only for complete deductions (cf. Alexander 169.17–170.16, 173.17–19); it seems more likely that 'complete' is intended only to apply to these two and that Aristotle has simply made an error.

Once again, Aristotle does not list all the deductions his principles imply: he gives *aii*, *eio*, *aoi*, and omits *aoo*, *aio*, *eii*, *eoi*, *eoo*.

33a26–27. 'It must be possible' (*anankē . . . endechesthai*): the 'must' is that which Aristotle regularly attaches to the conclusion of any deduction).

33a29. 'similarly related in position' (*tēi de thesei homoiōs echōsin*): that is, if the major premise is universal (as it was at 33a21–23).

33a31. 'evident': i.e., complete (cf. 24b24).

33a38–40. 'extending beyond (*huperteinein*) . . . predicated of equally many things (*ep' isōn*)': this language, with its stress on the extensions of terms, is relatively uncommon in the presentation of the theory of deductions (but compare its use in B 23, 68b23–29). The proof which follows may be regarded as a variant on *ekthesis*, a technique which also emphasizes the extensions of terms.

33a40–b3. The argument is as follows. Suppose first that A possibly belongs to some B (and therefore, possibly not to some B). It is consistent with this that A not belong (even possibly) to every B; suppose, then, that it does not, and let C belong to just those Bs to which A does not (even possibly) belong (which is consistent with the assumptions). Now, by hypothesis, it is not possible for A to belong to any C; therefore, both 'A possibly to every C' and 'A possibly to some C' are false. Moreover, by the equivalence of complementary converses, both 'A possibly to no C' and 'A possibly not to some C' are false. Thus, in the case imagined, *every* premise expressing possibility with A as predicate and C as subject is false, and thus no such sentence follows from the original premises. If the argument is to work, it is necessary to suppose (as I have) that C is the part of B to which A *necessarily* does not belong.

The argument is presented in a slightly convoluted manner. Aristotle first says 'let C be taken to be . . .' and then begins a clause with 'for' in which he spells out the proof: it is more natural to put 'for' at the beginning, as I have.

33b3–17. Aristotle's text suggests that he gives a countermodel proof by terms as an alternative. In fact, this argument is much more complete than the preceding argument, since it undertakes to show that a necessary and an assertoric conclusion also do not follow. Aristotle now returns to the double-triplet countermodels in application to modal deductions: here, the two 'conclusions' must be a *necessary* universal affirmative and a *necessary* universal negative. Aristotle takes some care in pointing out how such a countermodel rules out every conceivable conclusion: (1) the necessary affirmative rules out an assertoric or necessary negative conclusion, and the necessary negative likewise rules out an assertoric or necessary affirmative one; (2) the necessary affirmative rules out an *affirmative* possible conclusion (because 'what is necessary was not possible'), and the necessary negative similarly rules out a *negative* possible conclusion.

33b4. 'in the case of': what Aristotle means is that we can find premises of this sort in which the 'major' term cannot belong to any of the 'minor,' and *other* premises of this sort in which the 'major' belongs of necessity to all of the 'minor' (cf. 33b14–16).

Chapter 15

33b25–33. Aristotle here invokes, for the first time, a distinction between 'possibility according to the stated determination' (i.e., his usual sense, 'neither necessary nor impossible') and a looser sense of 'nonimpossibility.' As in the present case, he only appeals to this latter sense with respect to the conclusions of deductions. The weaker sense arises, in fact, only as the result of assuming a necessary premise in a proof through impossibility and deducing a contradiction from it.

34a2–3. 'contrariwise' (*enantiōs echontos*): that is, when the major premise is assertoric and the minor possible.

34a5–12. The purpose of the next section (34a5–24) is to clarify the workings of proofs through impossibility in the context of modal deductions. Aristotle's argument is complex and raises more issues than can readily be discussed here: for a related discussion, see *Metaphysics* IX.4. Throughout this passage, 'possible' is *dunatos* ('potential') rather than the usual *endechomenos* or *endechesthai*. Aristotle does sometimes appear to use *dunatos* quite like *endechomenos* (see the Note to 31b8–9), but *dunatos* is used in *Met.* IX.4 and probably has the sense 'potential' there.

34a12–15. Aristotle notes here that 'impossible,' etc., have application not only to states of affairs (*en tēi genesei*) but also to utterances (*en tōi alētheuesthai*) and to predication (*en tōi huparchein*). His point is that his preceding remarks about states of affairs may be extended to statements, in particular the premises and conclusions of deductions.

34a16–24. Although the point that nothing follows from a single premise is one Aristotle frequently makes, his goal here is not to rule out single-premise arguments, but rather to treat the premises of a deduction as a single thing so as to apply the argument of 34a5–12 to deductions. He does this by treating the premises much more like things (substances) than he typically does (this may explain his persistence in using *dunatos* rather than *endechomenos* throughout the passage). The final stage in the argument is perhaps more subtle than convincing, since it rests on labelling the premises with a single letter. 'A is necessary altogether' (*anankaiou tou A ontos hama*) as I translate it, means 'both the premises of which A is composed are necessary' (in Greek, where a neuter plural subject usually takes a singular verb, this is perhaps more readily acceptable than in English).

34a25–33. This passage gives the principle which Aristotle ultimately wishes to defend and on which his subsequent proofs rest: if a false but not impossible supposition is made, then an impossibility cannot follow from it. Aristotle's text, in fact, seems to make the stronger claim that a false but not impossible supposition *will* lead to a false but not impossible conclusion, which (as Aristotle takes some pains to show in B 2–4) is not the case. We might be able to save Aristotle from error by interpreting 'will be' (*estai*) as meaning 'will be possible' and translating 'it will be possible for what results from the assumption to be false, but not, however, <possible for it to be> impossible.' This is difficult grammar, however; Aristotle has probably been careless. In any event, the point he is really interested in, as the remainder of the argument shows, is that an impossible result cannot follow from asumptions which are not impossible.

Aristotle seems to overlook the case of premises which are individually possible but jointly impossible (such as 'every animal is awake' and 'some animals are asleep'). But he probably has in mind only the assignments of modalities to the various members of premise-pairs already known to yield deductions in their assertoric forms.

34a34–b2. This proof shows the application of the principle: Aristotle constructs a proof from impossibility by supposing both the contradictory of the desired conclusion and a premise *consistent with* one of the premises, and from this he deduces (as in the nonmodal cases) the contradictory of the other premise. But, in fact, his argument contains an equivocation. The assumption 'it is not possible for A to belong to every C,' if taken in the sense of possibility 'according to the determination,' is equivalent to '*Either* A of necessity belongs to some C *or* A of necessity does not belong to some C.' In some places, Aristotle takes note of this (see, for instance, 37a15–20). Here, however, he treats it as equivalent to the second disjunct alone: this is quite in accord both with ordinary Greek and with his own usage in the exposition of deductions with necessary premises (*to A ex anankēs ouch huparchei tini tōi B* and *to A ouch endechetai huparchein panti tōi B* may be used as equivalents), but to do so is to use 'possible' in the broader sense of 'not necessarily not.' As Ross points out, Aristotle should say here, as he does in the case of *eio*, that the only conclusion which follows from these premises is one of possibility in that broader sense.

34a40–41. 'it was assumed': the assumption, at 34a34, was 'A *belongs* to every B.' Waitz takes Aristotle to be making a tacit inference from this to 'It is possible that A belongs to B.' But such an inference only holds for the broad sense of 'possible.'

34b2–6. The sentence in brackets is found in all sources, but it is hard to get any coherent sense out of it. Ross rejects it as the work of 'a rather stupid glossator' (but conceivably, it is a garbled version of a direct proof).

34b7–18. This passage illustrates well how difficult it can be to make sense of Aristotelian modalities. The distinction between belonging 'at a moment' (*kata to nun*: compare *Physics* IV.11, VI.3) and belonging 'without qualification' or 'simply' (*haplōs*) is uncomfortably close to the difference between merely belonging and belonging of necessity (Alexander works hard to preserve a distinction: see 189.27–36). Conversely, if this passage does indeed contemplate a distinction between 'always' and 'necessarily,' then it appears to conflict with other passages (most prominently *On the Heavens* I.12) in which

Aristotle identifies them. In any event, Aristotle does not follow his own advice elsewhere, but is quite willing to use counterexamples 'according to time' in this way (see 35a20–24). (See Waterlow *1982*, Hintikka *1973* for extensive discussions of these points.)

34b22. 'let it not be possible': Aristotle immediately equates this denial of 'Possibly A to no C' with 'Necessarily A to some C' (compare 34b28–31).

34b31–35a2. The conclusion is 'not a possible one' in the strict sense, though it is in the broad sense; the deduction is 'of belonging to none of necessity' in that the conclusion *may* be necessary, though it need not be. Aristotle is evidently unhappy with his counterexample, perhaps (as Alexander suggests) because the major premises are not assertoric but necessary (or perhaps just because the two triples share only one term instead of his preferred two).

35a4. 'from the actual premises taken': the Greek (*ex autōn men tōn eilēmmenōn protaseōn*) could also be translated 'from the premises taken *themselves*.' However, Aristotle conceives of all completions of incomplete deductions as requiring the introduction of premises not actually taken but implicit in the premises that are taken (compare the similar uses of *autōn* at 37b31–32, 38a5–6, 38b33, 45a7).

35a5–6. 'converted accordingly': Aristotle says literally 'if the premise according to possibility is converted' (*antistrapheisēs tēs kata to endechesthai protaseōs*). But the phrase 'according to possibility' is his usual way of indicating *conversion* of the special sort applicable to possible premises. I have taken it here to be doing double duty, which is conceivable (cf. 35a14–15, 37b32–33, 38a32–33, 38b7–8).

35a8–20. Three times in these lines, Aristotle says that nothing follows through the premises as taken (once that there will 'in no way'—*oudamōs*—be a deduction), when what he means is that there is no *complete* deduction, though there is an incomplete one.

35a19. 'Which is true': that is, which follows by complementary conversion from the minor premise.

35a20–24. The countermodel triple used here takes 'White belongs to every animal' and 'White belongs to no animal' as possible premises: but since Aristotle regards some animals (e.g., swans) as necessarily white, others (e.g., ravens) as necessarily not white, these must be possible *at a time* in the way seemingly forbidden in 34b7–18.

35b2–11. This argument is a compressed but close parallel to that in 35a3–24 (even the countermodel triples for the nonmodal cases are the same). 'Through the indeterminate' has the sense it did in A 4–7: 'some snow is not an animal' is true because no snow is. Following Ross, I omit *kai* in 35b2; but 'or not belonging' (*ē mē huparchein*) in 35b4 is a harmless ambiguity. Even *kai* might be retained if regarded as explanatory ('There will be a deduction . . . *that is*, when the universal premise . . . ').

35b16. 'In alternation' (*enallax*) means 'one possible and the other assertoric.'

35b20–22. As Waitz points out, this summary is inaccurate: Aristotle rejects two combinations of possible major and assertoric minor (*ao*, *eo*) as nonconclusive.

Chapter 16

36a2. 'The same way as in the previous cases': presumably, the proof through impossibility used for *aaa* with possible minor and assertoric major.

36a7–15. Aristotle argues in this and a few other cases for an assertoric conclusion from a necessary and a possible premise. In each case, the proof is through impossibility and (as Alexander notes: 209.4–7, 216.28–32) turns on Aristotle's other, more celebrated modal curiosity: a necessary conclusion sometimes follows from one necessary and one assertoric premise.

36a15–17. According to Aristotle's strict definition of possibility, 'possibly does not belong' does *not* follow from 'does not belong.' We may charitably suppose that he means possibility 'not according to the definition' here, or we may uncharitably suppose that he has been inattentive.

36a17–18. This possible *e* premise and the necessary *e* of the previous example (36a8–9) nicely illustrate the potentially ambiguous expressions mentioned above in the Note on 31a6–7. The *only* difference between the statements of these two premises is the relative order of *mēdeni* and *endechesthō*: *to men A mēdeni endechesthō tōi B* (36a8–9), *to men A endechesthō mēdeni tōi B* (36a18). Waitz (397) and Alexander (136.23–29) recognize the potential these expressions have for ambiguity but do not note Aristotle's simple rule of word order.

36a21–22. 'premise from the side of the major extreme': a variant way of saying 'major premise.'

36b5. 'in relation to the major extreme' (*pros tōi meizoni akrōi*): Ross excises this phrase because it appears to say that the middle term in a first-figure deduction is predicated of the major extreme. Aristotle does make extremely frequent use of the preposition *pros* in this way (so that the predicate of a premise is *pros* its subject). However, he occasionally reverses this order in connection with major premises of first-figure deductions: exact parallels of the present case occur in the initial exposition of the assertoric first figure in A 4 (26a18–19, 26a39–b1).

36b19–21. 'in the case of belonging': that is, with *one* assertoric or necessary premise and one possible premise.

36b24–25. Although this sentence is in all the manuscripts, it contradicts Aristotle's claims that some of the deductions just considered are complete. As Ross points out, it is a verbatim copy of 39a1–3, which is correct in its place; it is, therefore, probably not due to Aristotle.

Chapter 17

36b35–37a32. Aristotle offers a lengthy proof (comprising three separate arguments) that 'it is possible that A to no B' does not imply 'it is possible that B to no A' (thus, possible *e* premises do not convert analogously to their assertoric and necessary counterparts). As noted above, the locution he uses is unfortunately ambiguous, being equivalent on one reading to 'of no B is it possible that it is A' and on the other to 'of every B it is possible that it is not A.' Interpreted in the former way, the sentence is, for Aristotle, a necessary universal negative and therefore converts in the same manner as a universal negative. Interpreted in the latter way, the sentence expresses possibility, and therefore, in the phrase of A 3 and A 13, is really affirmative in form: it is a universal affirmative and thus (as Alexander notes) does not convert.

36b37. 'let this be assumed': that is, assume that it *does* convert. Since this is the assumption of an argument through impossibility, I translate *keisthai* as 'assume': Aristotle occasionally substitutes the unprefixed *keisthai* for *hupokeisthai* in such contexts (e.g., B 12, 62a23; B13, 62b5).

36b38–40. 'contraries as well as opposites' (*kai hai enantiai kai hai antikeimenai*): as Alexander explains, this means: (1) 'possibly to every' converts with 'possibly to no' ('contraries'), and (2) 'possibly to some' converts with 'possibly not to some' ('opposites'). Alexander notes that these terms here indicate mere verbal form: the syntactical relationship between 'possibly to every' and 'possibly to no' is analogous to that between 'to all' and 'to no' (221.16–28). Despite what Aristotle says in B 8, 59b8–11, 'opposite' (*antikeimenos*) need not mean 'contradictory,' but only 'opposite' or 'opposed' in a generic sense.

37a2. 'this is incorrect': as at 30a27, the word is *pseudos*.

37a9. 'was not possible': that is, as determined in A 13.

37a10–14. Aristotle imagines an argument through impossibility which he thinks fails. I have inserted quotes to separate this putative argument from his comments: the 'for' clause beginning in 37a14 gives the reason why he thinks it fails, i.e., that 'not possibly to none' does not imply 'necessarily to some' but rather '*either* necessarily to some *or* necessarily not to some.' The awkward embedded double negative in 'it is not the case that if it is not possible for B to belong to no A' reflects an equally awkward construction in Greek (*ou gar ei mē endechetai mēdeni to B tōi A*).

37a15–17. 'used in two ways' (*dichōs legetai*): Aristotle does not mean that this phrase has two meanings, but that there are two different ways in which 'not possible to no' could be true (cf. the Notes on 27b20–23). This is one of the few places in which he takes express note of the fact that possible premises as he conceives them have (to use modern terminology) disjunctive negations. The negation of 'It is possible that B belongs to A' is really 'either B necessarily belongs to A or B necessarily does not belong to A.' To apply this to the quantified sentence 'It is possible that B belongs to no A,' we must first note that Aristotle considers this equivalent to 'It is possible that B belongs to every A.' This would be rendered false if there is some A (let us call it S) such that 'It is possible that B belongs to S' is false; and this, in turn, would be rendered false, either if S is necessarily B, or if S is necessarily not B. Since all that is important about S here is that it is some (part of) A, we can express the conditions for the falsehood of 'It is possible that B belongs to no A' as 'Either B necessarily belongs to some A or B necessarily does not belong to some A.'

But this last sentence is no longer a categorical sentence. In fact, since it is not 'one thing either affirmed or denied of one,' Aristotle probably would not regard it as a single statement at all. Whatever may be the case in that regard, he certainly makes no room for disjunctive sentences in the deductive theory of the *Prior Analytics*. Accordingly, what he does here is instead to say, in effect, that there are two ways in which the *negation* of the possible *e* premise *can be true*.

37a22. 'understanding it incorrectly' (*pseudōs an lambanoi*): not 'assuming a falsehood' but making an incorrect inference (so Tricot: 'commetrait une erreur'). Compare 30a27, 48a16, 49a18–22, and the associated Notes.

37a24–26. Since for Aristotle, 'possibly to none' and 'possibly to every' are equivalent, 'necessarily to some' and 'necessarily not to some' are each inconsistent with each of those. But the latter two are not equivalent: probably, what Aristotle is calling attention to here is that, in this case, we cannot find a one-to-one correspondence between premises and their 'opposites' (since we have two distinct components of the disjunctive negation, each of which is 'opposed' to the universal possible premise).

37a28–29. 'if this is taken': that is, if we take the *second* of the opposites ('of necessity does not belong to some') as the assumption of the attempted proof through impossibility which Aristotle has been discussing. The point can be put more precisely by going beyond Aristotle's analysis. If we want to get a strictly possible conclusion using a deduction through impossibility, then we need to assume the negation of the desired conclusion. But, as we have just seen, this assumption would need to be disjunctive ('either of necessity to some or of necessity not to some'). To make an argument through impossibility from such an assumption work, we would need to show that *each* of its disjuncts leads to an impossibility; and here, the second disjunct yields no contradiction. Once again, Aristotle's insight into the logical situation outstrips his own means for analyzing it.

In 37a28, I follow Rolfes and Tricot in reading 'not only . . . but also' (*ou monon . . . alla kai*), as in manuscripts B, d, and *n*, Philoponus, and Pacius, rather than Ross's 'not . . . but' (*ou . . . alla*).

37a35–36. The text here reads 'If we put it to be possible for B to belong to every C' (*tethentos gar tou B panti tōi G endechesthai*). The expected *reductio* hypothesis would be the *denial* of 'it is possible for B not to belong to every C.' Ross approves Maier's suggestion to insert 'not' (*mē*) twice: '*not* possible <for B> *not* to belong to every C.' I have inserted this in brackets into the text, but it should be regarded as tentative. Aristotle has just discussed at length the fact that the proper contradictory of 'possible not to belong to every' really contains two alternatives; it is not clear how he would reconstruct the putative argument through impossibility in view of that fact.

37a38–b3. What Aristotle gives here is a general rejection by countermodels of all combinations of two possible premises in the second figure. His language is more verbose and difficult than usual.

37b15. 'replace': i.e., replace them with other premises (compare 210a34, 56b8–9). The word *metalabein* can mean simply 'change,' and most translators so take it. However, elsewhere in the *Prior Analytics* Aristotle clearly uses it to mean 'put in place of,' 'substitute for' (see 39a27, 41a39, 48a9, 48a25–27). Here, it has the sense: 'substitute *something for*.'

Chapter 18

37b22–23. 'the demonstration is the same': that is, as in the case of two (universal) possible premises in the second figure (see 37a32–37). As before, the reason is that the possible *e* premise cannot be converted.

37b32–33. 'accordingly converted': cf. 35a5–6 and Note.

Chapter 19

38a21–22. 'will also not belong': a more literal translation of the Greek (*oud' huparxei*) might be 'will not belong either,' but this fails to bring out Aristotle's point, which is that we *also* get an assertoric conclusion.

38a30. 'it results that B of necessity does not belong to C': Aristotle presumably means that this *may* happen, just as any other categorical relation of B to C may happen (cf. 38a36ff).

38a36–38. Aristotle's appeal to the requirement that a necessary conclusion can be obtained only from two necessary premises or one necessary and one assertoric premise is illegitimate, since he has not offered any justification for that requirement (or for that matter even asserted it, except in application to the first figure). In the case of the earlier result that an affirmative conclusion only follows from two affirmative premises, Aristotle's proof was just an exhaustive survey of all cases, and there is no evidence that he has any other means of proof for this case: but he is engaged in just that survey here, and, thus, he can hardly appeal to its result.

38b3–4. The 'opposite affirmations' are 'B belongs/B belongs of necessity/it is possible for B to belong to every C'.

38b7–8. 'converted accordingly': cf. 35a5–6 and Note.
38b10. 'when the premises are converted': as Alexander points out (239.12–17), the major premise is converted by interchange of terms, the minor by complementary conversion. Aristotle omits 'of necessity' in the converted major premise.
38b13–14. Aristotle rejects any conclusion from two affirmative premises. But Alexander (240.4–11) offers a proof through impossibility that 'B is possible to no C' (in the wide sense) follows from 'A is necessary to every B' and 'A is possible to every C' and wonders why it is not acceptable. Probably, the reason is that Aristotle simply never accepts the deduction of a negative conclusion from affirmative premises. (Alexander makes similar remarks about other combinations of two affirmatives: see 240.32–241.1, 241.5–9.)
38b18–19. 'B may of necessity not belong': here 'may' is expressed with a future tense (*ex anankēs . . . ouch huparxei*).
38b21–22. 'has been shown' (*dedeiktai*): here the verb *deiknunai* does not mean 'prove,' (see the Note to 27b20), but rather 'exhibit' or 'show.' In 38b18–19, just preceding, Aristotle offered a counterexample 'showing' a term B of necessity not belonging to C; while this does constitute a proof that the relevant combination of premises and 'conclusion' is possible, it is not a proof that B of necessity belongs to no C.
38b34. 'converted accordingly': cf. 35a5–6 and Note.
39a3. 'through the aforementioned figures': Alexander argues that 'figures' here may mean 'deductions' ('moods'), since all the proofs Aristotle gives appeal to first-figure deductions. Ross suggests that Aristotle has in mind the fact that some of the first-figure deductions used in this section were earlier proved by arguments through impossibility that appeal to third-figure arguments with one necessary premise. Of course, this can be carried one step further: those deductions in their places were proved by appeal to still other first-figure deductions. All that Aristotle needs to be saying, however, is that nothing *other than* the aforementioned figures is required.

Chapter 20

39a11–13. 'we must take . . . similarly': that is, as before, we must sometimes take 'possible' in the conclusion in the strict sense, and sometimes take it in the wide sense.
39a27. 'if "is possible to belong" is substituted': i.e., if a universal affirmative is substituted for a universal negative (complementary conversion). Presumably Aristotle includes the two deductions which result if this is done either once (for the minor premise) or twice (for both premises).
39a29–31. That is, the deductions are exactly analogous to those with purely assertoric premises (except of course for the additional deductions through complementary conversion that Aristotle mentions at 39a38–b2).

Chapter 21

39b28–29. 'manner of the deductions will be the same': as in 39b9–10, this means that these deductions are analogous to their nonmodal counterparts.
39b31–39. Aristotle does not mention that the conclusion of this deduction will be possible only in the wider sense.
40a2–3. 'in the previous cases': the manuscripts all say 'in the universal cases,' which must be an error. Tredennick and Ross suggest that the reference is to the parallel treatment of cases with two possible premises (39b2–6). I follow Ross's suggestion of replacing 'universal' with 'previous'; in these chapters, Aristotle often says both 'as in the previous cases' and 'as in the universal cases,' so that the mistake is a natural one for a copyist to make.

Chapter 22

40a21–25. Aristotle's language is elliptical here, and I have tried to fill it out. I take *estai de palin to prōton schēma* to mean something like 'at this point, we get the first figure again *by conversion of the minor premise*'. As Ross notes, *gar* in 21 is 'anticipatory' ('since').
40a34–35. 'when the premise is replaced' (*metalēphtheisēs tēs protaseōs*): that is, replaced by its complementary converse. Compare the use of *metalambanein* at 37b15, 56b8.
40a37–38. Although Aristotle says the two term-triplets are for 'belonging to every' and 'belonging to no,' he must mean 'belonging *of necessity* to every/no.' The examples chosen are also curious. It is, of course, impossible for something to be awake or not asleep and be a sleeping horse. However, on Aristotle's usual understanding of necessity and possibility, no horse is of necessity awake or of necessity asleep: these are, instead, good examples of things that are 'capable of being otherwise' (in fact, Aristotle would elsewhere probably agree that every waking horse is potentially asleep and every sleeping horse potentially awake). The type of necessity involved here is actually closer to that which Aristotle regularly attributes to the conclusions of deductions: 'necessity when certain things are so' (*tinōn ontōn anankē*).

Chapter 23

This Chapter contains an extended argument that every deduction whatsoever can be transformed into a deduction resting only on first-figure universal deductions. As such, it is critical to Aristotle's overall project in the *Prior Analytics*. The argument picks up here precisely where A 7 left off: there is no trace of the account of modal deductions (this is evidence in favor of Bocheński's view that A 8–22 are a later addition to the work).
40b17–20. The opening claim of the Chapter has in no way been established for modal deductions. Aristotle proved many of the results in A 8–22 by

appeal to deductions other than *Barbara* and *Celarent*, and he identified a number of modal deductions as complete and thus not in need of proof. It is not at all clear how to apply the claim made here to these cases. We would, at the least, need an argument corresponding to that in A 7 to show how alternate proofs for these complete deductions could be constructed; such an argument would be quite complex, if possible at all, and there is no reason to suppose Aristotle had attempted it. Once again, the theory of modal deductions appears not to be well integrated with the rest of the text.

40b25. 'either probatively or from an assumption': on the term 'probative' (*deiktikos*), a term of art which may be an Aristotelian coinage, see also the remarks on 29a30–39. Aristotle contrasts probative deductions with those 'from an assumption' (*ex hupotheseōs*), and the latter include not only proofs through impossibility but also other types of arguments which, in his view, rest on assumptions. This contrast is related to a distinction in the *Rhetoric* between probative and 'refutative' (*elenktikos*) arguments (see *Rhetoric* II.22, 1396b22–27). Aristotle's views on the logical foundations of proof through impossibility are notoriously unclear and probably embody some confusions, even though he uses the procedure flawlessly in practice. See his own discussions immediately below, in 41a21–b5, and A 29, together with the associated Notes.

40b32. 'the initial thing' (*to ex archēs*, literally 'that from the beginning'), is the conclusion which it is required to prove. A basic rule of the game of proving is that one may not 'take' this 'initial thing' as a premise, but rather must obtain it by deduction from other premises. The phrase comes from dialectical practice: see B 16–17 and the associated Notes.

40b35–36. The claim that nothing follows from a single premise has indeed been *made*, and even *used*, before (A 14, 34a17–19), but it has not been proved in any way. Alexander says Aristotle must mean 'nothing follows *deductively*' (*sullogistikōs*) and appeals to the definition of *sullogismos*, in which Aristotle says 'certain *things* having been supposed (*tethentōn tinōn*), something else results of necessity': the plural then rules out single-premise deductions as a matter of definition (257.8–13; cf. his discussion of the definition and rejection of one-premise arguments as deductions, 17.10–18.7). Aristotle does not, in fact, believe that nothing can ever follow from a single premise, since he uses conversion inferences in his proofs of deductions: these must, therefore, fail to be deductions. Unfortunately, he never explains what the conversion rules are to be understood as, if not as deductions. Alexander's account, unsatisfying as it is, may be the best we can get.

41a1. 'connected' (*sunhaptēi*): Aristotle often uses this verb of a lengthy series of deductions (a 'sorites'), which eventually 'connects' a predicate term with its subject (as at 41a19, a few lines later).

41a2–4. 'there will not be a deduction . . .': Again, this claim is nowhere proved but seems to be a matter of definition for Aristotle. It should be noted, however, that the long discussion beginning in A 32 of how to get arguments into the figures at least serves to make plausible the claim that any argument can be put into figured form.

41a4. 'the kinds of predications' (*tais katēgoriais*): the 'predications' here are simply the types of categorical sentence. In the *Prior Analytics*, Aristotle usually uses the term 'problem' to express this.

41a7–18. The crux of Aristotle's argument is simply the requirement that a deduction have two premises which share one (middle) term. It then follows immediately from the definitions of the figures that every such argument must be in one of them.

41a18–20. 'The argument will also be the same': Exactly what this means is unclear. If we imagine a chain of predications, with *A* predicated of *C*, *C* of *C'*, and so on, ending with *B*, then we could plausibly claim that the whole series is in the first figure and joins up *A* with *B*. But the possible forms of such extended deductions are enormously variable: infinitely so, if we permit them to be arbitrarily long. A more thorough attempt to determine the possible forms of deductions is at least implied by *Posterior Analytics* I.19–22. In Smith *1986*, I argue that the project pursued in those Chapters may have been Aristotle's principal motive for developing the theory of deductions in the *Prior Analytics*; in Smith *1984*, I suggest that his project of classifying the possible structures of proofs recalls Hilbert's concept of proof theory.

41a22–b1. This discussion of arguments through impossibility should be supplemented by comparison with A 29, B 11–14, *Posterior Analytics* I.26, and *Topics* VIII.14. Aristotle undertakes to show that every such argument *contains* a deduction in the figures. Whether this suffices to prove his overall claim that all deductions whatsoever are 'both completed through the universal deductions in the first figure and led back into them' depends on how we interpret 'completed through' and 'led back into.' If it is taken to mean that every deduction can be *replaced by* a deduction containing nothing but first-figure universal deductions, then it is indefensible: Aristotle must treat the hypothesis of a deduction through impossibility as somehow external to the deduction.

Alexander (260.18–261.20) spells out the example given of proving the diagonal and side of a square incommensurable. The proof is found as the (spurious) last proposition (117) of the tenth book of Euclid's *Elements* (see Heath *1908*, III.2).

41a23–26. This description of arguments through impossibility appears to function as a sort of definition or canonical account: see 41a30–32. Note that Aristotle refers to arguments *coming to a conclusion* (*perainontes*) through an impossibility and says that they *prove* (*deiknuousin*) the intended conclusion when they *deduce* (*sullogizontai*) a falsehood or impossibility. According to the analysis presented here, an argument through impossibility is not, strictly speaking, a *deduction* of its intended conclusion, but only of the 'impossibility': the real conclusion is reached 'from an assumption' or 'from an agreement.' On such a view, we should not really speak of *deductions* through an impossibility, but only *arguments* or *proofs* through an impossibility (which will contain deductions *of* an impossibility). Aristotle appears to make some effort to conform his language to this, sometimes using the verb 'come to a conclusion' (*perainein/perainesthai*, as here and at 41a40; see also 51b2) or 'prove' (*deiknunai*, as at 41a25) rather than 'deduce.' However, he is not very consistent about this, and quite frequently reverts to speaking of deductions, or deducing, through an impossibility or from an impossibility (e.g., 45a26–27, 45b9, 61a18–18, 61a33, 62b25–26). The reason, no doubt, is that these expressions are part of the received technical vocabulary of his day, not his own coinage, and thus have an established usage towards which he inclines. See further the Notes on B 11–14.

41a30–32. 'For this is what': if the imperfect 'was' indicates reference to an earlier definition here, then this must refer to 41a23–26 (perhaps to the words 'when something impossible results when its contradiction is supposed'). The match is hardly a close one, however.

41a37. 'all the other kinds of deduction that are from an assumption': later, in A 28, Aristotle gives as examples of these 'those according to substitution or according to quality' (45b16–17). It is clear in that context that his views are not very well formed on this subject.

Chapter 24

41b7. 'belong universally': this includes *not*-belonging universally (i.e., belonging to none) as well.

41b7–13. In order to prove that there is no deduction without at least one universal premise, Aristotle could simply rely on his survey of all premise combinations in A 4–22 and observe that he has already shown every combination of two particular premises fails to yield a conclusion. What he actually gives us is a more complex argument which is both difficult to make sense of in its own terms, and hard to connect with the preceding account of deductions in the figures: I am inclined to think it is an older discussion of its topic which was composed in ignorance of the *Analytics*' theory of deductions. He presupposes a dialectical situation in which something is proposed for proof and one participant in the argument undertakes to prove it from premises obtained from an opponent by asking questions (see the Notes on 24a22–b15). Aristotle's claim is that if the person arguing does not manage to secure a universal premise, then one of three failings must attach to the argument: (1) there is no deduction, (2) it is not 'in relation to what was proposed,' (3) the person arguing will be 'asking for the original thing.'

His account is closely tied to the case of establishing a universal affirmative conclusion, and it is difficult to see how it should be generalized to other cases. Suppose that we are required to establish the conclusion 'Musical pleasure is good.' Aristotle then lists three mistakes we can make. First, we might try to get the premise 'Pleasure is good,' without adding 'every.' Aristotle says that this is an instance of case (1), although he does not explain why. Presumably, our attempted deduction would *not* have committed the fault in question had we taken the premise 'Every pleasure is good.' But we cannot deduce 'Musical pleasure is good' without taking some other premise in addition, and Aristotle does not indicate what that must be. What we need, of course, is 'Musical pleasure is a pleasure.' This might initially be taken to be a particular premise, but it is really universal, being equivalent to '*Every* [or *all*] musical pleasure is a pleasure.'

The second fault we could commit is to take some pleasure other than musical pleasure to be good. In this case, says Aristotle, 'it will not be in relation to (*pros*) what was proposed.' Does Aristotle mean that there may indeed be a deduction in this case but that it is not 'in relation to what was proposed,' or is he making a totally independent point? We may get some idea by considering what the premise taken in such a case would be like: it would be some premise of the form 'Such-and-such pleasure is good,' where 'such-and-such' is not 'musical.' But this is not really a particular premise, despite the fact that we could describe it as saying that some pleasure is good. Aristotle's way of describing the case suggests that he may, in fact, be thinking along these lines: saying 'if *it* is another' takes the word 'some' not as part of the premise supposed, but (in modern terms) as a metalogical term *describing* the premise taken. That is to say, if the premise taken is 'Mathematical pleasure is good,' then it would be correct to say that it had been taken as a premise that a certain pleasure is good, though not correct to say that the premise taken was 'A certain pleasure is good.'

The third fault is taking as a premise 'Musical pleasure is good,' which is, of course, the very thing we were supposed to be proving. Aristotle indicates that this is an instance of 'asking for the initial thing,' or in the traditional (and somewhat bizarre) translation associated with this phrase, 'begging the question.' For a fuller discussion of this criticism, see the Notes to B 16. Once again, Aristotle evidently regards 'Musical pleasure is good' as somehow not universal.

Alexander suggests that Aristotle's point is, not simply that every deduction must have a universal premise, but that every deduction must include a premise universal *in relation to* the subject term of the conclusion to be proved (266.20–31). If this means that every deduction must include a premise which affirms or denies something universally of its minor term, then it is simply false, as evidenced by *Darii*, *Ferio*, *Baroco*, *Festino*, and the entire third figure. However, it may well be close to what Aristotle has in mind. In many places, Aristotle talks as if every deduction were a first-figure universal deduction (this holds, for instance, in his discussion of extended deductions in A 25). It is at least possible that some of these discussions were originally composed before Aristotle had completely developed the theory of deductions in the figures. If that is the case here, then the overall point may be the claim that in every first-figure deduction of a universal conclusion, we must include some premise which affirms or denies something universally of the subject of the intended conclusion.

41b14. 'geometrical proofs': literally, 'drawings' or 'diagrams' (*en tois diagrammasin*). Aristotle often uses this term of geometrical proofs, which include diagrams. Similarly, the verb *diagraphein* ('draw out' or 'diagram') in Aristotle usually means 'prove by means of a diagram,' 'prove geometrically.' See the Notes on 46a8, 65a4–7.

41b14–22. There is some difficulty about determining just what geometrical proof Aristotle has in mind here: evidently, it is different from the proof of the same theorem found in Euclid (I.5). Alexander gives one reconstruction (268.6–24); Heath suggests that a pre-Euclidean proof, involving angles between straight lines and circles, may be preserved here. See Ross 374–376, Mignucci 429–430 for further discussion.

Whatever the actual proof is, the important problem is to determine what point Aristotle wants to illustrate. The salient factor, I think, is the use of a figure, which might be seen as a *particular* case. Thus, in reasoning about the figure in question, when we say such things as 'angle AC is equal to angle BD,' we must really take as our premise 'All angles of a semicircle are equal to one another.' He identifies three instances of such an error, and at the end says 'he will be asking for the original thing.' Since it appears that each of these is an instance of the same type of mistake, I have assumed that Aristotle means this last remark to apply to all three cases (so that he uses the geometrical example only to illustrate the *third* type of failing). Aristotle may be interpreting the role of the figure in a geometrical proof as somewhat analogous to what he describes in B 24 as an 'example' (*paradeigma*): see the Notes on that section.

41b24. 'both in this latter way and in the former' (*kai houtōs kai ekeinōs*): the 'latter way' is 'from all the terms being universal,' while 'the former way' may be taken as a rather loose reference to the beginning of the sentence ('must include belonging universally'), understood as meaning 'from at least one universal.' As frequently happens in the *Prior Analytics*, the meaning is clear enough but the grammar hard to explain.

41b27–31. The generalization offered here is rather complex. The traditional interpretation would be: (1) if the conclusion is affirmative, both premises must be; (2) if the conclusion is negative, one premise must be; (3) at least one premise must share the modal status of the conclusion. Since Aristotle thinks that an assertoric conclusion may be deduced from a necessary and a possible premise, (3) is inconsistent with what he has already said. The closing line is one of Aristotle's notes to himself to study a question further (cf. 50a39–40, 67b26, 69b38–70a2, and *Posterior Analytics* I.29, 87b16–18); since 'the other kinds of predications' are evidently the various modal relations, we have here further evidence that A 8–22 is later than the rest of A.

Chapter 25

In this Chapter, Aristotle considers for the first time extended deductions in which the conclusions following from pairs of premises may be subsequently used as premises for further conclusions. Much of what he says turns on the distinction between the 'main' (*kurios*) conclusion of an extended argument and the various intermediate conclusions in it. The Chapter is not fully consistent and may not have been fully worked out. Its purpose is almost certainly related to the argument in *Posterior Analytics* I.19–29 (especially 23, 29) concerning the possible structures of demonstrations: in 29, 87b16–18, we may have a demand for just the investigation we find here.

41b36–42a5. It is not fully clear what Aristotle wants to prove here: the assertion that every deduction is through only three terms, and thus two premises, appears to be a strong claim that every argument really rests on a *single* argument in the figures. Yet the Chapter clearly envisions extended deductions with intermediate conclusions, and at its end Aristotle counts up various possible structures of such arguments. It is possible that what Aristotle wants to show is this: whenever there are three or more *true* premises from which a conclusion follows, then there is necessarily some pair of *true* premises from which that conclusion follows.

Throughout this passage, Aristotle uses letters to stand both for terms and for premises. I have tried to leave the translation as ambiguous as (but, I hope, not more so than) the original in this regard.

41b37–42a1. The first exception Aristotle allows to his claim is the case in which the same conclusion follows with deductions having two different middle terms. Compare here *Posterior Analytics* I.29.

41b39. 'and also through A, C and D': the manuscripts give a wide variety of readings here: 'BC,' 'AC,' 'AC and BC,' 'BC and AC.' My translation follows Ross's conjecture 'ACD' (Aristotle often concatenates terms without conjunctions).

42a1–5. The second case is that of a genuine extended deduction. Aristotle actually distinguishes between deducing C from A and B, on the one hand, and deducing each of A and B from further premises, on the other. The sense of the argument must be this: Suppose that we have a case in which C is deduced from (say) D, E, F, and G, in the following manner: A is deduced from D and E, B is deduced from F and G, and C is then deduced from A and B. In such a case, we do not have a single deduction of C from four premises, but rather three separate deductions with three separate conclusions. On 'induction' (*epagōgē*) see B 23 and the associated Notes.

42a6–8. Aristotle concedes that his complex example might be counted as a single deduction but counters that, even if we have a deduction with more than three terms in that case, still the conclusion does not come about in *the same way* as C follows from A and B. The following argument (42a8–31) undertakes to spell out what that same way is. Evidently, Aristotle's real concern is to define what counts as a (minimum) single deduction; thus, as in the case of the claim that nothing can be deduced from a single premise, Aristotle's thesis may simply be true by definition. In 42a7, *dia pleionōn* must be elliptical for 'through more *than three*': cf. 41b36–37, *dia triōn horōn kai ou pleionōn*.

42a10. 'one as whole and the other as part': from the subsequent discussion, *A* and *B* are evidently premises and not terms. The relationship of whole to part for premises used here is not explained by Aristotle, though it is often taken as a reference to the *dictum de omni et nullo* of A 2 (24b28–30). The phrase 'this was proved earlier' is presumably a reference to the argument just given that every deduction must include 'universal belonging' (41b7–27). But as we have seen, the interpretation of that argument and the exact determination of the claim it is intended to establish are problematic, and thus the relationship to the present passage is also uncertain. It might be, instead, that Aristotle means to refer to the entire account of deductions in the figures.

42a23–24. 'induction, or concealment': Aristotle regularly distinguishes *epagōgē* and deduction (cf. *Topics* I.12), which allows him to ignore such arguments for his present purposes. 'Concealment' (*krupsis*) means adding extraneous matter to an argument to make it harder for one's opponent to detect one's purposes. Ross appropriately refers to *Topics* VIII.1, 155b20–24.

42a24–30. As this passage makes clear, Aristotle's strategy is to show, for every putative case of a conclusion from more than two premises, *either* that it is not a single deduction, *or* that the additional premises are deductively superfluous.

42a34–35. 'taken in addition for the purpose of completing': this fits particularly well with the view that Aristotle's concept of completing a deduction is a matter of supplying the necessary steps to get from the premises to the conclusion.

42a35–40. The 'main' (*kurios*) conclusion of an extended deduction differs from the 'upper' or 'anterior' conclusions (*ta anōthen*) in that it is not also used as a premise for a further conclusion. Similarly, Aristotle later refers to those premises which are not also 'upper' conclusions as the 'main' premises (42b1). The claim that the main premises must be even in number is puzzling: not only does it not follow from what Aristotle has said, but also it seems to be contradicted by 42b5–16 (where it is said that the number of premises is odd if the number of terms is even, and *vice versa*). Possibly, what Aristotle has in mind is this: Call a *one-layer* deduction a deduction with two premises. Call a *two-layer* deduction the result of replacing *each* premise of a one-layer deduction with a one-layer deduction of that premise; and, generally, call an *n-layer* deduction the result of replacing each of the highest-level (main) premises of an $(n - 1)$-layer deduction with one-layer deductions of them. In such a deduction, there is always an even number of main premises (and indeed, an n-layer deduction has 2^n main premises). Aristotle ignores or neglects complex cases in which main premises may be at different heights.

42a38. 'must be premises': note that *protasis* here indicates an argumentative role (thus, to translate 'proposition' would be seriously misleading).
42a39–40. 'for its position' (*pros tēn thesin*): that is, for the purpose of defending the statement he is required to defend in the exchange.
42b1. 'Counting deductions by [*kata*] their main premises': that is, *counting* or *individuating* deductions by reference to their main premises.
42b4–5. The claim that the conclusions will be half as many as the premises is still more puzzling. In an n–layer deduction, there are half as many 'upper' conclusions at the second highest level as there are main premises (and, in general, half as many conclusions at each level as at the one above); but there are 2^{n-1} conclusions (including both the main conclusion and all intermediates). Aristotle's remark is true strictly only in the case of a one-layer deduction, with two premises and one conclusion.
42b5–26. Aristotle contrasts 'prior deductions' with 'continuous middles.' The latter is related in sense to the notion of deductions 'joined up' (*sunhaptoi*) with each other found in previous sections: each member of a continuous series of middles is predicated of its successor (the case Aristotle has principally in mind is a series of terms each of which is universally true of the next). Arguments with 'prior deductions' are presumably those with intermediate conclusions. It is not clear just how Aristotle conceives these two as related, or what extent either has, since the subsequent discussion entirely concerns continuous terms. The term 'prior deduction' (*prosullogismos*) is not common in Aristotle, but the sense is apparently not confined to the context of extended deductions: see *Topics* VI.10, 148b4–10.
42b6. Note that Aristotle uses letters here to stand for terms, and not (as in 41b36–42a40) premises.
42b8. 'the term inserted': Aristotle distinguishes adding a term to one end of a continuous series from adding a term in the middle. Einarson *1936* argues that the term 'inserted' (*parempiptōn*) probably derives from Greek proportion theory.
42b9–26. To understand Aristotle's argument, suppose that we have a continuous series of terms $A_1 \ldots A_n$ where for each i, A_i is predicated of A_{i+1}. There are then $n - 1$ 'intervals' between adjacent terms in this series, and, thus, we have a series of $n - 1$ 'continuous' premises. From any pair of adjacent premises in this series, we can deduce another premise; this yields another series of $n - 2$ adjacent premises. Proceeding thus through $n - 1$ iterations, there are $(n - 1) + (n - 2) + \ldots + 1 = (n^2 - n)/2$ conclusions. Adding a term to the series, in effect, adds one premise at each of the $n - 1$ levels and moves the original conclusion down one level: thus, the number of conclusions is increased by n, the *initial* number of terms.
42b16–18. 'the conclusions will never have the same arrangement [*taxis*]': all this seems to mean is that there is no fixed *ratio* of number of conclusions, either to number of premises or to number of terms. Again, it is not clear why Aristotle is interested in this. He may have in mind some parallel with Pythagorean *gnōmon*-arithmetic: cf. *Physics* III.4, 203a13–15.

Chapter 26

42b27. 'what deductions are about' (*peri hōn hoi sullogismoi*): Ross explains this as 'what syllogisms aim at doing, viz., at proving propositions of one of the four [categorical forms].' But the phrase has a close parallel in *Topics* I.4, where Aristotle tells us that 'what arguments are from' (*ex hōn hoi logoi*) and 'what deductions are about' are 'equal in number and the same.' He then explains that arguments are 'from' premises and 'about' problems (*problēmata*). The term 'problem' in the *Prior Analytics* means 'type of categorical sentence' (as here, at 42b29; see the Note on 26b31). In the *Topics*, however, both 'premise' and 'problem' carry with them more of a suggestion of argumentative role. As the associated verb *proballein* ('throw out [as an obstacle]'), suggests, a *problēma* in dialectic is the proposition under discussion (which one party to the debate undertakes to defend and the other to attack). In *Top.* I.4, 101b28–36, Aristotle tells us that a premise and a problem differ only in the way they are presented (*tōi tropōi*), so that by a change in this, a problem can be converted to a premise, and conversely. Scholars differ on just how to interpret this, but it seems to me that Aristotle has in mind the fact that the same proposition (to use a modern term) might figure in two argumentative roles: as a premise to argue from or as a 'problem' for debate. In the latter case, Aristotle suggests, it might be expressed in a form beginning with 'whether' (*ara*). This implies that a problem is a two-sided question; but Aristotle also defines a dialectical premise as an 'asking of a contradiction,' so that there is a natural correspondence of the two.

I think we can make a guess as to how 'problem' came to have the sense it does in the *Prior Analytics*. First, we should note that the problems in dialectical arguments actually have less to do with the construction of the arguments themselves than premises: premises are the sources which deductions are built from, whereas problems would only serve to indicate what conclusions one should be aiming at. However, certain features of a problem do matter for the dialectical debater: as the present sections of the *Prior Analytics* make clear, knowing which type of categorical sentence it is influences how one goes about looking for an argument to establish or refute it. Consequently, it is important to determine the *kinds* of problems. It is easy to imagine Aristotle abbreviating 'kinds of problems' into 'problems' with repeated use: compare the more famous abbreviation of 'kinds of predicates' (*ta genē tōn katēgoriōn*) into 'predicates' (i.e., 'categories').

42b29. 'easy to approach' (*euepicheirētos*): this word has as its root *epicheirein*, 'lay hands on,' which elsewhere in Aristotle means 'attack' in dialectical contexts (cf. Note on 24b10–11); here, that is nearly reversed. The sense of the word is 'easy to lay hands on.'

42b30–31. 'in more figures and by means of more cases' (*en pleiosi schēmasi kai dia pleionōn ptōseōn*): it seems clear enough that 'case' (*ptōsis*) means 'mood,' in traditional terminology. (In the *Topics*, arguments from 'cases' are those relying on parallel substitutions of certain inflected forms: see especially II.9.) Aristotle has no settled way of saying 'mood': most commonly, when referring to deductive patterns, he does so through metalogical comments about the figures, e.g., 'we saw that in the first figure when the major premise was universal and affirmative . . .'

43a10. 'more cases': Ross reads 'more ways' (*tropōn*), and his apparatus implies that this is the reading of all his sources; Waitz reads 'cases' (*ptōseōn*), and his apparatus implies that *this* is the universal reading. According to Williams, Ross simply reports the manuscript testimony incorrectly. Nothing is very surprising about 'case' appearing here in this sense, since it has just been used in 42b30. However, it is at least of some historical significance if *tropos* does not occur here. The word 'mood' (as a technical term in connection with syllogisms) ultimately descends from *tropos* used in this sense (as in the Greek commentators), by way of the Latin *modus*. If *ptōseōn* is the right reading here, then it appears that Aristotle himself never referred to a mood as a mood.

Chapter 27

43a20–24. Several details of Aristotle's vocabulary in this sentence merit comment. 'Being supplied' (*euporein*) is the opposite of 'being at a loss' (*aporein*). The reference to a 'route' or 'way' (*hodos*) recalls the beginning of the *Topics*, which sets as the goal of that treatise 'finding a way of pursuit (*methodos*) by means of which we may be able to deduce about any problem proposed from things accepted' (100a18–22); the goal of discovering a 'way' is an old one in Greek philosophy. The term 'principle' (*archē*) literally means 'beginning': its sense here is that found in the *Posterior Analytics*, where principles are the first premises on which all scientific demonstrations depend. The expression 'the principles concerning any particular subject' is a common one in Aristotle, especially in the *Posterior Analytics*. Aristotle often distinguishes between 'common principles,' which serve among the principles of many or even all sciences, and the principles 'peculiar' or 'proper' to each given science. This doctrine represents a rejection of the Platonic view of all science as forming a unity resting on a single set of highest principles (or even a single highest principle). The details of Aristotle's views on this subject are much too complex to enter into here, but it is probably in order to mention that by the 'principles concerning any particular subject' Aristotle could mean the principles *peculiar to* that subject.

43a25–43. Aristotle's procedure rests on a division of 'things that are' (*ta onta*) into three classes: those which are subjects of predication but never predicates, those which are predicates but never subjects, and those which can be both. The first class is exemplified by Aristotle as 'the individual and perceptible' (*to kath' hekaston kai aisthēton*), though he does not here address the question whether it includes other types of things as well. The second class is regularly identified by commentators as the categories, but, in fact, Aristotle says only that he will 'explain later' (*palin eroumen*) that 'it comes to a stop at some point proceeding in the upwards direction' (*epi to anō poreuomenois histatai pote*). The phrase 'it comes to a stop' almost certainly has the quasi-technical sense it does in *Posterior Analytics* I.3 (72b11, 72b22) and I.19–22 (81b32–33, 36; I.20, 82a22; I.21, 82a36–37, 82b11–12, 25-28, 32, 35; I.22, 83b30, 39–84a1, 84a28, 39–b1), where it is associated with his argument that there are premises not susceptible of proof; he establishes this by arguing that there cannot be an infinite chain of predicates each of which has a higher predicate true of it. (Thus, the chain 'comes to a stop eventually.')

The third class consists of everything else. Curiously, having made what appears to be an important distinction, Aristotle promptly disregards the first two classes. One explanation, offered by Łukasiewicz and Patzig, is that the rules of conversion can be given unrestricted scope only if every term is able to function both as subject and as predicate, which only holds of terms in the third class.

(Concerning this section, see also the Note on 65a10–25.)

43a26. 'predicated of nothing else truly universally' (*kata mēdenos allou katēgoreisthai alēthōs katholou*): here, 'universally' is not a term of quantity (opposed to 'particular') but has its metaphysical sense (opposed to 'individual'). In Aristotle, the term 'universal' functions grammatically as an indeclinable noun, adjective, or adverb, as the context may require. I have taken it as adverbial, but it could conceivably be adjectival ('truly predicated of nothing else universal'). The phrase 'truly universally' (*alēthōs katholou*) means 'genuinely as a universal,' not 'universally true' or 'truly and universally.' Perceptible individuals are not merely not predicated of anything *universally*, they are also not predicated of anything *at all* (except 'incidentally,' as Aristotle says). Mignucci comes close to this sense by translating *katholou* as 'absolutely' ('assolutamente'), but unfortunately I do not think Aristotle ever uses the word in this way.

43a34–35. Being predicated 'incidentally' (*kata sumbebēkos*) is illustrated well enough by Aristotle's examples. For a discussion of the meaning of this problematic expression, see Barnes *1975*, 118–119.

43a40. 'other things are predicated of them': the Greek is just 'these of others' (*tauta kat' allōn*), which could equally well mean 'other things are demonstrated to be predicated of them.'

43b1–11. Aristotle's procedure for finding principles consists in collecting all the premises one can find about a given term S and classifying their predicates into three groups: those which follow S, those which S follows, and those which are inconsistent with S. Obviously, the selection is to be made from among *true* premises. What Aristotle gives us is a way to take a collection of all the truths about some subject, and then determine both what can be proved from those truths and how to construct those proofs. In the terminology of modern logic, his method is comparable to a *decision procedure* for deductive systems. It has been suggested that the procedure is ultimately derived from

Plato's *Phaedo*; but even if this should be true, Aristotle's justification of it in the following Chapter is fundamentally dependent on his theory of deductions. (See the Notes on 46a17–27.)

As later becomes evident, the selection process defined here is to be applied to both the subject and the predicate of the proposition one wants to prove. Aristotle refers to the term to which the process is being applied as the *pragma*, a word with a rather loose range of meanings; I translate it 'subject' in the sense 'subject of discussion,' but it might also be rendered as 'thing.'

43b5–6. 'need not be selected' (*ouk eklēpteon*): this could mean either 'need not' or 'must not,' but I think the former better fits the context.

43b6–8. 'predicated . . . essentially' (*en tōi ti esti*: literally, 'in the what it is') may roughly be translated 'predicated in the definition': see *Posterior Analytics* I.4. 'Peculiar' predicates (*idia*) are those true coextensive with a given subject. This division resembles the fourfold division given in the *Topics* (I.4), where it is asserted that every predicate true of a subject is true of it as its definition, its peculiar property, its genus, or its incidental characteristic.

43b9–11. This sentence contrasts 'more quickly hitting on the conclusion' (*thatton entunchanesthai sumperasmati*) with 'demonstrating more' (*mallon apodeiknunai*). According to the *Posterior Analytics*, true premises are a necessary condition for demonstrating at all, and at any rate, demonstration does not seem to admit of degrees: Aristotle's phrase might be taken to mean 'demonstrate more often' (i.e., produce more demonstrations). My rather unattractive translation tries to preserve the unclarity.

43b19–20. 'we also propose premises': the Greek is 'we propose' (*proteinometha*). The verb *proteinein* ('stretch out,' 'hold out,' 'offer'), is cognate with 'premise': a premise is 'that which is held out' (i.e., for acceptance or rejection in an argument). Noun and verb are closely associated in the *Topics* (for a good picture of the relation, see 104a3–7), but this is one of the relatively few appearances of *proteinein* in the *Prior Analytics* (for another see 47a15). Its association with *protasis* is strong enough that the noun may be supplied (the only thing Aristotle ever speaks of proposing is a premise). The phrase 'both useless and impossible' (*achrēston . . . kai adunaton*) embraces both a methodological point (such relationships of terms contribute nothing to the search for premises) and a syntactical point (sentences like 'every man is every animal' are not grammatically well formed).

43b22–32. The rationale behind the various restrictions given in this section on what predicates should be selected is evident once we see the purposes to which it is put. In discussing the relationships of selections for a term contained under another and that term containing it, Aristotle refers to the containing term as 'the universal' (*to katholou*): his language is compressed and difficult. The reference to 'those which do not belong' means 'those which do not belong *to any*.'

43b24. 'need not be selected' (*ouk eklekteon*): as at 43b5, this fits the context better than 'must not.' The subject of 'have been taken' is 'the things following or not following the <containing> universal.' These *need* not be added to the selection of terms which follow the contained universal because they will already have been included. To use Aristotle's example, if we are selecting terms in connection with 'man,' then all those which follow 'animal' will automatically be selected among those which follow 'man.' Similarly, whatever 'does not follow' (is inconsistent with) animal also 'does not follow' man in this sense.

43b36–38. 'Things which follow everything' are terms such as 'being' (*on*). This practical recommendation recalls the view common in the logical works that there is neither a genus nor a science of everything. Aristotle gives the reason why in 44b20–24.

Chapter 28

Aristotle now proceeds to spell out the procedure for finding true premises from which to deduce a given proposition. In fact, in the course of his argument he argues not only that his procedure can find deductions if they are possible, but that it is the *only* procedure needed.

43b39–44a11. Aristotle's presentation of his method recalls certain features of his proofs of deductions in A 4–22. He first states the result which he is going to prove: here, that result consists of a set of rules for finding premises from which to deduce a given conclusion by looking for common terms in the sets defined by the procedure of A 27. As in the earlier proofs, these results are stated entirely without benefit of letters. In highly abbreviated language, a proof of the result follows (44a11–35), in the course of which Aristotle introduces letters.

44a11–17. Aristotle uses letters here, not to stand for terms, but to stand for sets of terms. However, in the course of his proofs, he treats these letters as a sort of formal predicate: B, for instance, is used to mean 'a term from class B.'

44a17–35. Aristotle now shows that each rule is correct by showing how to construct a deduction with the desired conclusion in each case. Some details of his reasoning suggest that this passage antedates the account of A 4–7. He presents a third-figure deduction of a particular affirmative conclusion (*Darapti*) with essentially no justification (44a19–21). By contrast, in the treatment of universal negatives, he spells out a completion through conversion of second-figure *Cesare*, calling it an argument 'from a prior deduction' (*ek prosullogismou*) (44a20–24). The subsequent discussion of second-figure *Camestres* (44a25–27) makes no mention of conversion. At no time does he appeal to what was established earlier in the account of deductions. The deductions given for particular conclusions are both third-figure (*Darapti*, 19–21; *Felapton*, 28–30); this is a result of the procedure, which admits only universal premises into the selected sets, but the treatments of these cases resemble the proofs through *ekthesis* in A 7. Aristotle also adds what amounts to a fourth-figure deduction (*Bramantip*) in 30–35, calling it a 'converted deduction' (*antestrammenos sullogismos*).

It is significant that Aristotle's procedure makes use only of universal premises. Since he later claims that it is also sufficient to find any premises for a deduction that can be found, he must hold that anything that can be proved at all can be proved from only universal premises. This can be shown to be equivalent to the two assumptions on which the procedure of *ekthesis* rests.

44a38–b5. Here Aristotle makes yet another use of letters. He continues to use *A*, *C*, *E*, and *F* as they are defined in 44a11–17, and '*KC*' evidently indicates (in modern terms) the *union* of *K* and some particular member of *C*. Such a clearly extensional conception of terms is unusual in the *Analytics*. The point Aristotle wants to make is obscure. Ross suggests that he is recommending the choice of that middle term which is widest in extension of all possible middles. This may be correct, but: (1) if '*KF*' is true of every *E*, then it should already have been counted among the terms in *F*; (2) it is quite unclear how this could be applied to anything but universal affirmative deductions (this is a difficulty with many of Aristotle's remarks: see the Notes on A 24).

44b4–5. The 'first terms' here are not the 'primary' terms of A 27, 43a25–43, but simply the first terms (in order) *which the subject follows*, i.e., those predicates which it implies. In Aristotle's usage, 'A follows B' is equivalent to 'B is below A.' The terms below those which follow a term are then the terms which the terms it follows follow. (And since 'follows' is a transitive relation, the terms following the term in question also follow these terms.)

44b20–24. Here Aristotle fulfills the promise made in 43b36–38.

44b25–37. Aristotle now gives an elegant proof, based on his treatment of deductions, that identical terms occurring in any pairs of groups other than those treated never yield a conclusion.

44b38–45a22. Aristotle closes with an argument that his procedure comprehends all that is worthwhile in any alternative ways of searching for premises which also take account of pairs of contrary or different terms among the various term collections. Relations of contrariety, in particular, were important in the milieu of the Early Academy, and contraries play a major role in Aristotle's own *Topics*. It is likely that his remarks are directed at some actual set of procedures advocated by others, or even by himself at an earlier stage.

His argument is again elegant, resting on two claims: (1) we must look for something the *same*, since what we are looking for is a middle term, and the middle term is what is the same in the two premises; (2) in any event, whenever it is possible to prove something because of relations of contrariety, Aristotle's method will also discover a proof.

45a9–16. This passage as it stands is seriously confused. The case Aristotle is considering is one in which some term *B* that belongs to every *A* is the contrary of some term *G* that *E* belongs to all of. In this case, since *B* and *G* cannot belong to the same thing and *B* belongs to every *A*, *G* therefore cannot belong to *A* and is thus identical to a member of the class *D*; as a result, Aristotle's method would discover this case too. However, what the text says is not '*G* will be the same as one of the *D*s', but '*B* will be the same as one of the *H*s,' which, if true, would yield a deduction that *A* is true of *no E*. The justification of this offered in lines 13–16 seems to make no sense. Ross brackets the entire passage as an addition by 'a later writer who suffered from excess of zeal and lack of logic,' but if we do so we are left with no treatment for this case. (In 45a12, some manuscripts have 'but to no *G*' rather than 'to no *E*,' and one source has 'but *E* to no *H*.' I do not see how any combination of these can make more sense.)

45a17–22. 'carrying out a superfluous examination': this is a somewhat expansive rendering of the verb *prosepiblepein*, which Aristotle uses only here. 'A different route from the one needed' (*allēn hodon . . . tēs anankaias*): *anankē* here means 'necessary' in the sense 'that without which not' (cf. *Met.* V.5, 1015a20–26). Compare the reference to a 'different route' at 46b24.

Chapter 29

One important difficulty remains for Aristotle's claim that his procedure will find a deduction if and only if a deduction is possible: arguments through impossibility. He maintained earlier in 41a22–b1 that every such argument, insofar as it is deductive, must consist of an argument in the figures. Here, he makes a much stronger claim: every argument through impossibility can be replaced by a 'probative' argument, and vice versa. The consequence would be that proof through impossibility is a completely redundant process. But in A 45, Aristotle seems, instead, to be arguing that there is no alternative to proof through impossibility as the means of completing second-figure *Baroco* and third-figure *Bocardo* (see the Notes on 50b5–9, 51a40–b2).

In the present argument Aristotle really wants to show that whenever a *proof*—a deduction from true premises—through impossibility exists, then there must exist true premises from which a probative deduction of the same conclusion could be constructed. He argues for this by treating the assumption in a proof through impossibility as *independently known* to be false, and then constructing the probative deduction by using the contradictory of that assumption, which must, therefore, be true. But this cannot be applied to the technique of *completing deductions* through impossibility, where the result is not a consequence that contradicts an independently known truth but rather a straightforward inconsistency (assuming *p*, *q*, and *r*, we deduce the contradictory of *q*).

We might try to harmonize Aristotle's views by attributing to him a comparatively sophisticated distinction between proofs through impossibility at the level of the deductive theory itself, and proofs through impossibility relying on deductions established in that theory (see the Notes on 27a14–15). In my opinion, however, it is equally likely that Aristotle's views are simply not fully worked out. See, further, the discussion in B 11–14 and the (perhaps badly confused) remarks in *Posterior Analytics* I.26.

As in A 23, Aristotle follows his treatment of arguments through impossibility with a few general remarks about extending the case to all types of arguments 'from an assumption.'

45a29–b8. Aristotle assumes that every deduction through impossibility must use the contradictory of the intended conclusion, together with one of the premises, to deduce an 'impossibility,' i.e., a statement known to be false. But given the established structures for deductions, it follows that this impossibility must be a statement containing the middle term and that one of the extremes which is not found in the premise used in the original deduction. The contradictory of this 'impossibility' must then be true, and therefore the middle term must fall into one of the three term-classes defined for that extreme. Since we already have a premise about the middle and the other extreme, we need only combine these to produce a deduction; moreover, Aristotle's method is sufficient to have found this deduction in the first place. Aristotle further claims that the process can be reversed to generate a deduction through impossibility wherever there is a probative deduction.

This argument does nothing to establish that the technique of *completion* through impossibility is redundant. At most, it shows that, given the full complement of deductions Aristotle has established, it is always possible to make use of one of these deductive forms directly, rather than constructing a full proof through impossibility.

45b6–7. 'converted': that is, *negated* (one of the many meanings of *antistrephein*: cf. B 8–10). The premise in question is actually the 'impossibility' deduced, not one of the premises from which it is deduced.

45b8–11. Given Aristotle's way of understanding proofs through impossibility, his claim that one of the premises of such a proof is 'put falsely' is correct, provided that we take the premises to be *all* those statements from which the deduction is constructed (the preceding sentence seems to indicate this). Thus, Aristotle regards the deduction through impossibility as really having three premises: the two found in the corresponding probative deduction, together with the denial of the intended conclusion.

45b12–20. As in A 23 (41a37–b1), Aristotle ends his argument with the claim that it can be extended to every kind of argument 'from an assumption.' The two examples he gives, arguments 'according to substitution' (*kata metalēpsin*) and arguments 'according to quality' (*kata tēn poiotēta*) are not discussed elsewhere in Aristotle under those names. The commentators explain the former as arguments resting on a conditional assumption of the form 'If *p*, then *q*,' where *q* is the thing to be proved and *p* is the 'thing substituted.' The 'thing substituted' is a substitute *subject for argument*: one wants to prove *q* and so, getting a concession that if *p* then *q*, one 'substitutes' *p* (i.e., makes it the subject of discussion). Compare the analogous passage in A 23, 41a38–b1, where Aristotle says that in *all* these types of arguments 'the deduction comes about in relation to what is substituted' *ho sullogismos ginetai pros to metalambanomenon* (and see also *Top.* II.5, esp. 112a21–23). Arguments 'according to quality' are explained by the commentators as resting on principles typified by

'If this, then *still more likely* that' (as we call them, arguments *a fortiori*). In the *Topics* and the *Rhetoric*, Aristotle frequently lists several types of arguments 'from more and less and likewise' (*ek tou mallon kai hētton kai homoiōs*); the identification is plausible, though not certain. Although we find a subsequent brief discussion of arguments from an assumption in A 44 (with which this passage should be compared), 45b19–20 here shows that Aristotle realized his theory, and even his system of classification for such arguments, was in need of much further work.

The phrase 'when we come to discuss proof through an impossibility' presumably refers to B 14.

45b21–28. Aristotle briefly considers another possible exception to his procedures, somewhat similar to the case of pairs of contrary terms among the term-classes discussed in 44b38–45a22. Suppose that E belongs *only* to Gs and that G is a member of class C (i.e., the things of which A is universally true). It follows that E belongs only to things of which A is universally true. To get from this to the conclusion 'A belongs to every E' as Aristotle does requires a principle something like 'If E belongs only to what A belongs to universally, then A belongs to every E'. It is not at all clear either how to interpret such claims or how to accommodate them to Aristotle's theory of deduction. Apparently similar sentences are found in B 5–7 (58a29–30, 58b9–10, 58b37–38, 59a28–29); later commentators called these deductions through *proslēpsis*, though there is no Aristotelian authority for that use of the term, and no clear indication that Aristotle recognized such a class of arguments. (On this point see the notes on B 5–7.) Given his overall argument, we would expect Aristotle to tell us here how these arguments also may be brought within his procedure; instead, he seems to indicate that they fall *outside* it. It may be that this is a more than usually unfinished note about a problem case which Aristotle never resolved.

This procedure may also have some connection with Aristotle's account of induction: see the Notes on B 23.

45b23. 'by means of the examination for a particular' (*dia tēs kata meros epiblepseōs*): the procedures Aristotle indicates look for common terms in classes C and G or D and G; in the basic procedure as defined in 44a11–35, these are the term-class pairs to be checked when trying to prove particular affirmative or negative conclusions respectively.

45b28–35. As at 49b29–31, these very cursory remarks about possible and necessary conclusions and 'the other kinds of predication' suggest that the arguments of this section were completed before Aristotle had worked out the contents of A 8–22. The only deductions envisioned are those in which the premises are both of the same modality as the conclusion, rather than the complete study of all combinations of modalities given earlier. (And what he says here seems to imply that there are second-figure deductions with two possible premises, contradicting A 17.)

Cases of a predicate which does not belong but is nevertheless capable of belonging, which Aristotle is careful to include here, would actually add nothing at all to the theory of A 8–22.

45b32–34. 'for it was proved': It is quite unclear what Aristotle has in mind here as having been proved, and where he supposes himself to have proved it. Alexander (329.17–29) supposes that Aristotle means to distinguish cases in which a statement is actually false but possibly true (which he calls 'genuinely possible,' *kuriōs endechomenon*) from those which are both true and possibly true; he takes the reference to be to Aristotle's characterization of possiblity in A 13, 32a18–21. Ross says that 'this was shown in the chapters on syllogisms with at least one problematic premise' (that is, the whole of A 14–22); though he does not explain what 'this' is, he evidently takes it to be that a possible conclusion can follow from premises, not all of which are possible. But what Aristotle actually says is that in selecting attributes one should include those which *do not belong but are capable of belonging*; and the treatment of deductions with at least one possible premise in A 14–22 never takes any account at all of this point. Mignucci's suggestion (p. 462) about the meaning of the passage is probably the best: Aristotle is simply noting that in the case of possible premises, we must include in class B, for example, not only things which follow A, but also things which are capable of following it, but do not. But the reference of Aristotle's 'it was proved' is still obscure.

Chapter 30

Chapters 30–31, which provide the grand conclusion to the account of the method for finding proofs that began in A 27, must be understood against the background of Plato's views on 'dialectic' and the proper method for philosophy. Plato accepted two views which Aristotle vehemently rejects: (1) there is a single set of principles from which all the truths about reality may be derived; (2) the procedure of 'division' (as presented in the *Sophist* and the *Philebus*) is the proper method for finding these principles. In addition, Plato held that knowledge of the principles is somehow innate and can be recovered in a way akin to remembering. Aristotle frequently denies that there is any single set of principles, and in some places he links this with denials that there is any single correct approach appropriate to all areas of inquiry. In this Chapter, however, he rather grandiosely offers us his procedure as 'the route' to be followed in all areas whatever. He takes care to make clear just how this is related to his rejection of a universal science, and stresses the role required for observation and the collection of facts. In a closing *tour de force*, he attacks the method of division by arguing that, to the extent that it is of value, it is already included (as 'only a small part') within his own procedure, and that, as a result, he is in a better position to understand it than its own practitioners.

46a5. 'discern' (*athrein*): for the sense of this verb, see *Metaphysics* III.3, 998b1, and *On the Heavens* II.13, 293a29. A passage in the pseudo-Aristotelian *Problems* brings out the meaning well: 'those who are drunk cannot discern distant objects' (872a19).

46a8. 'things that have been strictly proved to belong' (*ek tōn kat' alētheian diagegrammenōn huparchein*): this might be rendered 'things that have been *diagrammed* to belong according to truth.' Translators generally suppose it to mean something like 'from an *arrangement* of terms in accordance with truth' (so Jenkinson). But that would evidently be just an obscure periphrasis for 'from true things.' Aristotle never makes the absurd supposition that different patterns of argument are *valid* in scientific and in dialectical contexts respectively, and in fact he often insists on the reverse. Moreover, he never refers to the terms in a deduction or figure as *diagegrammenoi*. The correct sense comes from the use of *diagraphein* and *diagramma* in connection with geometrical theorems: see, for instance, 41b14, where *diagramma* can only mean 'geometrical proofs.' A striking passage to compare is *Metaphysics* V.3, 1014a35–b3, which refers to 'what are called the elements of diagrams or of proofs in general.'

Aristotle contrasts strict proofs with 'dialectical deductions,' indicating that the basis of the distinction is the epistemic status of the premises on which each rests. Such a view is found elsewhere (for instance, *Topics* I.1, 100a27–b23). However, Aristotle appeals to a different sort of criterion in 24a16–b15 (see the Notes).

46a17–27. Aristotle now recalls his doctrine that each science rests on its 'peculiar principles' (*idiai archai*, *oikeiai archai*), with no overarching, general principles from which all scientific knowledge can be derived. He says '*the majority* are peculiar' here probably to take account of the 'common principles' (*koinai archai*), which he exemplifies most frequently by the law of excluded middle and certain generalized mathematical claims, that may figure into many different sciences. The status of these common principles is problematic for him: in several places in the logical works (e.g., *Posterior Analytics* I.11, *Sophistical Refutations* 11, *Rhetoric* I.2) he seems anxious to dismiss these as not really principles, at least not in their general forms, but in *Metaphysics* IV he argues that at least the principles of noncontradiction and excluded middle are genuine principles of a science of being as such. This issue is too complex to discuss here.

Aristotle here gives us an especially clear picture of just what his method amounts to. We get the principles of a science by means of experience and his method, as follows. First, experience gives us the 'facts about any subject' (*ta huparchonta peri hekaston*), that is, the collection of all the truths about it. These data constitute a 'collection of facts' or 'history' (*historia*) concerning the subject. We then use this summary to draw up the various term-classes with respect to each term in the science. Application of the procedure to any truth in this *historia* will then yield premises from which to deduce it, *if they exist*. If they do not exist, then (as Aristotle points out here) the procedure will also make that clear. The latter point is of great significance in the light of the theory of demonstration in the *Posterior Analytics*. Aristotle defines a demonstration as a deduction the premises of which meet a number of qualifications, among them being true, 'primary' (*prōtos*) and 'unmiddled' (*amesos*). It is clear from what follows, especially *Posterior Analytics* I.19–22, that this latter term means 'without a middle,' that is, 'lacking a middle term by means of which it can be proved.' It is a peculiarity of Aristotle's deductive theory that a collection of truths (such as an Aristotelian science) may, and indeed under certain fairly general circumstances must, contain statements not derivable from any other combination of statements in the set. Such 'unmiddled' statements cannot be demonstrated, since they cannot even be deduced from other true statements; they must, therefore, be among the principles of the science (*epistēmē*) corresponding to the initial *historia*. (See also the Note on B 16, 65a10–25.)

46a18. 'it is for our experiences concerning each subject to provide the principles' (*tas men archas tas peri hekaston empeirias esti paradounai*): this phrase is grammatically ambiguous, since *empeirias* could be either genitive singular or accusative plural. In the first case, the clause would mean 'it is for experience to provide the principles concerning each subject.' But in the very next sentence, Aristotle gives an example of this, and the relevant experience appears in the accusative case: *tēn astrologikēn empeirian*. Given Aristotle's great propensity for the ellipsis of repeated elements, I venture to infer, both that the verb 'provide' is understood in this second case, and that *empeirias* in the first case is also accusative. Aristotle thus stresses, not just that experience provides us with knowledge of the principles of sciences, but that experience *of the relevant subject matter* provides us with the principles concerning that subject matter.

46a27. 'make that evident' (*touto poiein phaneron*): that is, 'make that *situation* evident,' to wit, that the statement in question cannot be demonstrated. Nothing in Aristotle's preceding discussion explains how his method could make indemonstrable facts *themselves* (as opposed to their indemonstrability) evident. However, his method could indeed make it evident that a statement is not susceptible of proof (if the *historia* and its associated *eklogē* are genuinely exhaustive of all the facts concerning the subject). The question how we come to understand the principles of sciences is discussed—with celebrated obscurity—in *Posterior Analytics* I.19 (though Aristotle's answer is already hinted at in 46a17–22).

46a28–30. The reference is normally taken to be to the *Topics* (perhaps I.14). But there is no discussion in the *Topics* that can plausibly be called a 'more detailed account' of what Aristotle has just gone through, and the discussion of 'how premises are to be selected' in *Topics* I.14 seems very remote from the present subject. Since this sentence intrudes into an otherwise fairly cohesive line of argument in A 30–31, I suggest it may be a later editor's attempt (inspired by merely verbal similarity) at tying together the two passages.

Chapter 31

This Chapter is complementary to *Posterior Analytics* II.5, which evidently refers to it (91b12–14).

46a31. 'Division by means of kinds' (*hē dia tōn genōn dihairesis*) is the procedure given in Plato's *Sophist* and *Statesman*. The person dividing begins by determining some overall genus within which the thing to be defined is included. This genus is then divided into two (or perhaps more) subgenera, and the *definiendum* is assigned to one of these. The process is repeated until a last genus is reached which is identical with the *definiendum*. Aristotle is sometimes criticized for treating this procedure as a rival in some way to his theory of deductions: after all, it may be urged, the two have quite different objectives, and therefore it is no more a valid criticism of division that it fails to prove than it would be appropriate to object that deductions or demonstrations fail to define. But there is a deeper point. Aristotle's real complaint is that division is not a method which leads to the acquisition of *knowledge*: at each step, the divider must 'ask for the initial thing.' Thus, the method cannot produce understanding.

46a32. 'procedure' (*methodos*): to translate this as 'method' obscures the connection with *hodos* ('route'). Aristotle uses *methodos* as a synonym of *hodos* as in 43a21, 45a20, 45b37, 46a3, and later at 46b24, 46b33. Elsewhere, *methodos* often means 'scientific discipline': see the Note on 53a2.

46a36–37. A 'demonstration concerning substance, or what something is' would be a proof of a definition. Aristotle's own views on this subject, as expressed in *Posterior Analytics* II.5–10, are somewhat difficult of interpretation (see Ackrill *1981*), but he seems to reject the notion that definitions or essences are subject to proof.

46a37–39. Aristotle charges the partisans of division with two errors: not only did they miss the true procedure (his), but also they failed even to understand their own.

46b5–7. Here we have a deduction with a disjunctive middle term ('B or C'). Aristotle does not tell us much about how he understands such terms, but there is some evidence available in *On Interpretation* 8 about the comparable case of conjunctive terms.

46b20–22. 'take the universal . . . their extremes': the 'universal' is the term Aristotle has designated A, the 'differences' are the two contraries with which it is divided (B and C), and 'that about which' is the term to be defined (D).

46b24. Those who divide 'follow out their different route in its entirety' (*tēn allēn hodon poiountai pasan*): that is, carry their procedure all the way to its end. (Compare the reference to 'another route' at 45a21). 'Solutions' translates *euporiai*, which is opposed in this sense to *aporiai*: compare *Metaphysics* III.1, 995a29, *On the Heavens* II.12, 291b27.

46b26–37. For good measure, Aristotle notes several other things his deductions can do which division cannot: refute, argue about things other than definitions, and give us knowledge we do not already have. According to the *Posterior Analytics*, demonstrative deduction accomplishes the latter not by showing us new facts, but by giving us explanations.

Chapter 32

Aristotle now begins the last major project of *Prior Analytics* A: explaining how any argument may be put into the forms of the three figures. This project is somewhat comparable to translating natural-language arguments into a formal language. As a result, this section (A32–A44) consists mostly of heuristic devices and explanatory notes, though there is some important theoretical material, especially in 44. (A 45 and A 46 do not fit so clearly into Aristotle's announced goals: see the Notes on them below.) Much of *Prior Analytics* B (especially 23–27) may be a continuation of this project (again, see the discussions below in the Notes).

46b40–47a2. Aristotle promises to tell us how to 'lead back' (*anagein*) or 'dissolve' (*analuein*) existing arguments into the figures. These two verbs share the prefix *ana*, which suggests either motion upwards or returning back to some starting point. Aristotle has already used *anagein* of the process of completing a deduction, and later in A 45 he uses it of a somewhat more general process of transforming deductions from one figure to another by means of conversions. *Analuein* appears to be synonymous with *anagein*, though its root meaning would suggest decomposing arguments into their consituents in some way. In *Posterior Analytics* I.12, 78a6–8, it appears to designate specifically the search for the premises from which a conclusion can be proved. Since the title 'Analytics' (*ta analutika*) is Aristotle's own and applies to the *Prior* and *Posterior* as a whole, these processes are evidently a primary concern of his.

47a2–5. The summary given here corresponds well to the divisions Aristotle himself indicates in the text of Book A: the first part comprises at least 1–22 (and perhaps 1–26), and its completion is indicated at 43a16–19; the second is announced at 43a20–24 and comprises Chapters 27–30; and the third, which begins here, is declared complete at 51b3–5. But it is possible that much of the material in Book B could also be attached to this third section (see the Notes on B, particularly B 16, B 22–27).

47a5–9. The declaration with which this passage closes is reminiscent of Parmenides.

47a12–13. 'larger parts': that is, premises (rather than terms). "Universal': in Greek, actually 'in a whole' (*en holōi*).

47a18. 'other useless things' (*alla de matēn*): compare 42a23–24 on 'concealment.'

47a21. '*asked* in this way' (*houtōs erōtēmenous*): Aristotle clearly has a dialectical situation in mind (cf. 24a24–25, 24b10–12).

47a31–40. Recent interpreters have taken this section as evidence that

Aristotle recognized that some valid arguments were not 'syllogistic.' But given the broad generality of application he has tried to show for his account, it would be odd to find such a concession here; and given his definition of *sullogismos*, it is difficult to know what it would mean. In its context, the passage probably has a much less profound sense: 'cases like these' must refer to the examples discussed in the preceding lines, which Aristotle has characterized as arguments in which some premise has been left out. Therefore, he is only saying that some (persuasive?) arguments fail to be deductions as they are presented because certain obvious premises are left unstated. In any event, he does not say that they cannot be accommodated by his methods, but rather, that one should not try to resolve them 'straight off' (*euthus*).

47b1. 'predicated and a subject of predication': in Greek, this is the same verb in active and passive voice (*katēgorēi kai katēgorētai*). Aristotle almost never uses *katēgorein* like this: the passive is normally used with the sense the active voice has here, and the active normally used only of a person uttering a predication.

Chapter 33

In this Chapter Aristotle makes the sensible point that not everything which appears to be arranged as a deduction in the figures really is so. The specific error he is concerned with is usually identified as that of taking an indeterminate premise to be universal. However, the examples he uses to illustrate this point appear, instead, to make it more obscure: each involves a proper name, and Aristotle complicates matters by presenting us with premises in which quantifiers are applied to proper names. Interpreters have, therefore, found the Chapter a source of considerable difficulty; my Notes aim more at indicating what the points of obscurity are than at resolving them. For an insightful recent discussion, see Bäck *1987*.

See Ross's notes for possible identifications of Aristomenes and Mikkalos (who appear in Aristotle as examples only in this passage).

47b21–29. Aristotle locates the problem in his first example in the fact that the (putative) major premise 'Thinkable Aristomenes always is' is indeterminate (lacks a quantifier). However, he says, if we add a universal quantifier to this premise, then it becomes false. Why does Aristotle call it false, rather than nonsensical (as we might be inclined to)? The answer may lie in the fact that the Greek *pas* (like French *tout*) may be used both with sortal terms (like English 'every') and with singulars and 'mass terms' (like English 'all of' or, in the same sense, 'all'). As a result, *pas ho dianoētos Aristomenēs aei estin* could also mean 'All *of* thinkable Aristomenes always is,' which is false for perishable Aristomenes. (Aristotle may have taken note of this distinction: cf. *Metaphysics* V.9, 1018a3–4.)

Mignucci, taking note of this last point, suggests instead that 'Aristomenes' in the second premise is a general term meaning 'person named "Aristomenes.'" Thus, 'Thinkable Aristomenes always is' would mean 'Of the thinkable [conceivable?] men with the name "Aristomenes," there will always exist at least one.' This is an ingenious suggestion; but if that is what Aristotle had in mind, why then does he say 'But this is false . . . if Aristomenes is perishable'?

47b23–24. 'Thinkable Aristomenes always is' (*aei gar esti dianoētos Aristomenēs*): while it is usually taken to mean 'Aristomenes *qua* thinkable always exists,' Aristotle just might have in mind also the sense 'Aristomenes is always thinkable.' I have tried to keep the ambiguity in my translation. Indeed, there may even be a sort of pun involved: *dianoētos* can mean 'intended' or 'what is meant,' so that this sentence might mean 'It is always Aristomenes that is meant.'

47b29–37. Aristotle thinks his second example commits the same fallacy as the first. The reason why 'Musical Mikkalos will perish tomorrow' is not 'universally true' might be that this *sentence* is not always true (if true today, then it was not true yesterday). Or, if the suggestion offered for the last case is plausible, then *ou gar alēthes katholou, Mikkalos mousikos hoti phtheiretai aurion* might mean something like 'it is not true of *the whole of* musical Mikkalos . . .', i.e., part of musical Mikkalos (the plain unmusical man) will remain. Of these two possibilities, the first seems to me more likely: the rather odd grammar of the sentence (word for word, 'it is not true universally, musical Mikkalos that he will perish tomorrow') resembles the sort of construction which in Aristotle often functions like a pair of inverted commas.

Once again, Mignucci takes the argument to involve a use of proper names as a sort of general term: 'musical Mikkalos will perish tomorrow' could mean 'some musician with the name Mikkalos will perish tomorrow,' and of course it does not follow from this that our particular musical Mikkalos will perish tomorrow.

Chapter 34

48a2–15. Aristotle's first example of terms 'not well set out' is the deduction 'Health necessarily belongs to no illness; every man is susceptible of illness; therefore, health necessarily belongs to no man.' He solves the difficulty by substituting terms 'applying to the conditions' (*kata tas hexeis*), i.e., attributive terms like 'healthy' and 'ill,' in place of the corresponding 'conditions' (*hexeis*), i.e., abstract singular terms like 'health' and 'illness'. The minor premise then becomes 'Healthy necessarily belongs to nothing ill,' which Aristotle says is false.

Aristotle's example seems to be a modal deduction, with the minor premise equivalent to 'It is possible for every man to be ill' and the major premise and conclusion necessary. According to A 16 (36a7–15), the conclusion from this combination of premises should be *assertoric*: Aristotle ignores this, and instead concedes that a *possible* conclusion might be acceptable. The problem about the setting out of terms really concerns the interpretation of abstract singular terms in predications: 'Health belongs to no illness' might be taken as a statement about two Platonic universals, as a statement about the classes of healthy and ill things, or in some intermediate way. There may be important connections here with Plato's questions about the 'communion of kinds' in the *Sophist*: see Vlastos *1972*, *1973a*.

48a15–18. Aristotle extends his treatment to a parallel second-figure case ('Health cannot belong to any illness; health can belong to every man; therefore, illness can not belong to any man'). His account is cryptically brief: presumably, he thinks it is parallel to the previous example. (If so, he again contradicts his earlier treatment: cf. A 19, 38a14–25.)

48a16. 'mistake' (*pseudos*): this word usually means 'falsehood' or 'false.' However, immediately preceding at 47b40 the verb *diapseudesthai* clearly means 'make a mistake,' not 'state a falsehood' or 'lie.' Compare *Nicomachean Ethics* VI.6 1141a3, 1144a35; *Metaphysics* IV.3, 1005b12, and the Notes on 30a27, 37a22, 48a16, 49a18–22.

48a18–21. Aristotle is less clear about what the third-figure case is supposed to be. Ross thinks it is 'Health can belong to every man; illness can belong to every man; therefore, health can belong to some illness.' The apparent paradox is then eliminated by the suggested substitution, so that the conclusion becomes the unproblematic 'Some healthy things are possibly ill.'

48a21–23. These lines (whether Aristotle's or an editor's) reflect second thoughts about the consistency of the doctrine just stated with the rest of Book A. The reference appears to be to 39a14–19, where second-figure *Darapti* with two possible premises is said to yield a possible conclusion.

Chapter 35

48a29–31. 'with a <single> word' (*onomati*): the term *onoma*, 'name,' is used by Plato and Aristotle very much like 'noun' as a grammatical term, but unlike 'noun' it has the ordinary sense 'name' and sometimes the broader sense 'word'.

48a31–39. The error Aristotle identifies here is not, as generally supposed, believing that demonstration of an unmiddled statement is possible, but mistakenly taking a specific deduction to have unmiddled premises. Otherwise, Aristotle's example is irrelevant: he offers us, not an apparent deduction of an unmiddled statement, but a deduction with an apparently unmiddled *premise*. It is clear that the minor premise of the deduction 'Everything isosceles is a triangle' is unmiddled, since it is a matter of definition. The difficulty is that 'Every triangle has <the sum of its internal angles equal to> two right angles' appears to lack a middle term, even though it is demonstrable (it is, in fact, Aristotle's favorite example of a geometrical theorem; as usual, he states it in an extremely elliptical fashion). This discussion recalls *Posterior Analytics* I.5, which refers to cases of a 'nameless' (*anōnumon*) middle term (74a8) and discusses the same geometrical example.

48a35–37. 'possesses two right angles of itself': in *Posterior Analytics* I.4, the expression 'of itself' (which is an Aristotelian technical usage) is explained as equivalent to 'essentially,' i.e., 'in the definition'; but in the following Chapter 5 it is evidently treated as equivalent to 'just as' or '*qua*.' With this in mind, Ross takes Aristotle to mean that 'there is no wider class of figures to which the <two-right-angle> attribute belongs directly.' But this implies that in a demonstration the middle term must always be *wider* than the minor, which is contradicted elsewhere in the *Posterior Analytics* (cf. I.13, where all three terms of a demonstration are coextensive). The fact is that this is an especially hard case for Aristotle: there is no very plausible way to get the demonstration of this theorem Aristotle knows (cf. e.g., *Metaphysics* IX.9, 1051a24–26) into figured form. Ian Mueller *1974* argues (in my opinion convincingly) that in general, Greek geometrical arguments strongly resist being recast in Aristotle's canonical forms for very fundamental reasons. In the present case, Aristotle must say that there really is a middle term but that it can only be expressed in a 'phrase' (*logos*), though he gives us no idea what that 'phrase' might be (the Greek term would even admit the senses 'sentence' or 'discourse': Ross's strained attempt to supply a middle term shows how extended that phrase may have to be).

Chapter 36

48a40–b2. Although Aristotle generally treats 'belongs to' and 'is predicated of' (or 'is said of') as synonyms, here he distinguishes them, evidently taking the former to be wider in extent than the latter. His claim is that the relationship of predicate to subject in a deductive premise need not be one of predication, though it must be one of 'belonging.' It is clear that Aristotle wants to extend the range of application of his deductive theory to cases which, as he recognizes, cannot easily be treated as categorical sentences. However, it is less clear how he proposes to do that. Mignucci (480–481) proposes that Aristotle here restricts 'predicated' to cases in which the subject term is in the nominative case, while leaving 'belongs' with its customary wide sense in which it can indicate 'any possible grammatical construction for a predicative relation.' Ross (407) says that 'we must take account of the cases of the nouns and recognize that these are capable of expressing a great variety of relations and that the nature of the relations in the premises dictates the nature of the relations in the conclusion.'

These do seem to be descriptions of what Aristotle is trying to do. But it is not at all clear how such an extension is to be justified. Aristotle says that he rests his entire deductive theory on definitions of 'predicated of every' and 'predicated of no' (24b28–30). If he now wishes to extend the notion of a premise to include cases in which the 'predicate' term is *not* predicated of the 'subject,' then what becomes of this theory?

Aristotle does not tell us anything about this, but his examples suggest that he is now willing to admit premises which in some respect *behave like* strictly categorical sentences, even though they are not. Thus, it is the formal relationships among premises considered as sentences that he is calling attention to, rather than the semantic basis of those relationships in the relationships of what their terms denote. It is tempting to see a parallel here to another passage. In *Posterior Analytics* I.5, Aristotle offers the theorem that proportional terms are also proportional *alternando* (i.e., if A:B::C:D, then A:C::B:D) as an example of a theorem that formerly was proved as a different result in different ways about different subject matters but 'now' is given a 'universal' proof covering them all (74a17–25). Before the 'universal' proof was discovered, there were several formally similar statements about proportional numbers, lines, solids, or times; the formal similarity among these eventually led to the development of a single proof embracing them all. Similarly, Aristotle here may be trying to increase the generality of his theory of deductions by exploring other types of premises formally similar to the categorical sentences with which he began.

48b2–4. Despite appearances, there is little if any connection between this sentence and the famous doctrine of the homonymy of 'is' as found in *Metaphysics* IV.2, 1003a33–b15, VII.1, 1028a10–31, XI.3, 1060b31–1061a10.

48b4–7. The example treats the sentence 'There is a single science of contraries' as having as predicate 'there is a single science *of them*' (*mian einai autōn epistēmēn*) and as subject 'things contrary to each other' (*ta enantia allēlois*). Ross takes the predicate to be 'that there is one science,' supposing that Aristotle thinks this is affirmed of 'contraries' by saying, of contraries, that there is one science of them. But this cannot be right. Aristotle has just explained that 'belongs' has as many senses as 'is' here, so that, presumably, we could construct the requisite premise with 'is.' But this would give us something like 'contraries are there being one science,' which is nonsense. Instead, Aristotle takes the predicate to have included in it 'of them' (*autōn*). Thus, what is said to belong to contraries is, not that there is a single science, but that there is a single science *of them*. There are several objections that can be raised about this analysis from the standpoint of modern logic, but an important point that Aristotle does *not* miss (and which Ross apparently does) is that the predicate in question is *relational*: what is being said about 'contraries' is not 'there is a single science' but 'there is a single science *of those contraries*.' Aristotle's device of building this into the predicate may not be very well worked out, but it shows that he was sensitive to the problem.

48b7–9. 'Not in the sense that contraries are a single science of them' (*ouch hōs ta enantia to mian einai autōn epistēmēn*): this is the received Greek text in most sources. One of the oldest manuscripts (*n*) and Georgios' Syriac translation add 'are' (*esti*) after 'contraries,' which supports my rendering more explicitly, though the same sense can readily be gotten without it. Ross finds this unintelligible, and changes the text to *ouch hōste ta enantia mian einai epistēmēn*: 'not so that contraries are one science.' But Aristotle tells us that the *right* way to understand this premise is as saying 'there is a single science *of them*' about contraries, and we should, therefore, expect the same predicate-term in the first case. It is perfectly in order, in fact, for the first clause to be nonsense: Aristotle's purpose is to argue that the sentence in question is *not* a predication but something else, and to do that he shows that taking it as a predication results in a nonsensical reading. Mignucci (481) gives an interpretation in some ways similar to mine.

Note that even though he claims that it is not a predication, Aristotle says that 'it is true to say' its predicate-term of its subject. This suggests that he is not completely sure how he wants to carry out his own analysis here.

48b10–27. We now have three examples in which one or both premises are not really predications: Aristotle says one term is not 'said of' (*legetai*) or 'predicated of' (*katēgoreitai*) another. In every case, the conclusion can be interpreted as an assertion of existence, and such statements are quite difficult to put into Aristotle's required canonical form.

48b10–14. Aristotle's first example of a deduction with nonpredicative premises is clearly intended to be in the first figure. It is perhaps a little clearer how it is supposed to work if we stay close to the Greek word order:

Wisdom is <a> science (*hē sophia estin epistēmē*)
Of the good is wisdom (*tou agathou estin hē sophia*)
Therefore, of the good is <a> science (*tou agathou estin epistēmē*)

Aristotle simply presents the example and observes that neither the minor premise nor the conclusion is a predication. He does not, however, explain how we should understand these premises, if they are not predications, and he does not try to argue that the deduction is a valid one. He defends his claim that the conclusion is not a predication with the observation that 'the good is not a science': this suggests that he takes the minor term to be 'good,' not 'of the good.' The oblique case (in this case genitive) must, therefore, be part of the relevant sense of 'belong' here, so that one way in which X can belong to Y is for X to be 'of' Y. From this, we might suspect that Aristotle is now using 'belongs' in an extremely wide sense, almost like 'is in some relation to.'

One problem with this example is that it is difficult not to take the minor premise and conclusion to be existential ('there is a science/wisdom of the good'). This may not be Aristotle's intent. Ross avoids the existential sense by rendering the deduction as 'wisdom is knowledge, the good is the object of wisdom, <therefore> the good is an object of knowledge.' But this seems to me too highly interpretative to be a translation: the existential reading should not, I think, be foreclosed, and in any event, 'object of knowledge' is very hard to see in Aristotle's Greek.

48b14–19. Aristotle's next example is something like 'Science is of every contrary or quality; the good is a contrary, and a quality; therefore, science is of the good.' He points out here that both in the minor premise and the conclusion, 'science' is not predicated of anything, although it is in the major premise. Given the discussion in 48b5–6, we might expect him to say that the major term is not 'science' but something like 'there being a science of it.' However, throughout this discussion Aristotle evidently assumes that any expression containing the *word* 'science' must really be an occurrence of the *term* 'science,' regardless of the construction.

48b20–24. We next get a deduction in which there is no predication at all. It is perhaps significant that both 'science' and 'genus' are relational terms here (a science and a genus are respectively the science *of* something and the genus *of* something).

48b27. 'not said of one another': that is, the middle term is not said of the last nor the first of the middle.

48b27–49a5. Aristotle now says he will turn his attention to negative cases. His examples are all in second-figure form, probably because the 'predicate' must apply to two different subjects in the same oblique case (and so he needs the premises to share a predicate-term). 'There is no motion of motion' (*ouk esti kinēseōs kinēsis*) seems to be a straightforward negative existential statement. 'There is not a sign of a sign' could be a sort of grammatical remark ('We do not speak of a sign of a sign'), but the parallel with 'There is a sign of laughter' (*gelōtos esti sēmeion*) should incline us towards an existential reading. 'There is opportunity for a god' (*theōi kairos estin*) probably has to be taken as existential.

As before, Aristotle takes terms really to be expressed by their nominative forms, even when they do not occur in this way at all in the argument (as in the example, 'There is opportunity for a god; nothing is useful (and / or needed) for a god; therefore, opportunity is not <the time> needed,' in which, despite the fact that 'god' occurs only in the dative case, Aristotle says the term to choose is the nominative form).

48b34–35. 'through the genus being said about it': commentators beginning with Alexander take this to mean, in effect, 'in the second figure.' The 'genus' is simply the middle term (which, since it is predicated of both extremes, is perhaps analogous to their 'genus). 'About it' (*pros auto*) apparently means 'about the problem,' i.e., about *each* of its terms.

It is worth taking note here of Aristotle's grammatical terminology because these are among the earlier occurrences of such technical language in Greek. He uses the term 'inflection' (*ptōsis*) to designate any inflected form derived from an adjective or noun stem by changing the ending. Our use of the word 'case' in its grammatical sense ultimately descends from this Greek word (by way of the Latin *casus*). However, for Aristotle the nominative form is *not* an inflection: he refers to it, instead, as the 'appellation of the noun' (*klēsis onomatos*). Moreover, included among cases are not only what we would count as oblique cases (genitive, dative, accusative), but also adverbial forms with the ending *–ōs*.

49a2–4. The first three of Aristotle's examples are of relational terms complemented by the dative, genitive, and accusative cases, respectively; the last is just a *non*-relational predicate in the nominative case, here exemplified, not by a predicate in isolation, but by a predicate ('animal') used in an example.

Chapter 37

This little note proclaims its own incompleteness. I translate *katēgoriai* as 'predications' on the basis of parallels with 41a4, 41b31, 45b34 (the sense 'categories' is certainly not appropriate). As in the latter two passages, Aristotle probably has in mind modal qualifications; the call for further study then, presumably, is partly answered by A 8–22. 'Either simple or compound' (*ē haplas ē sumpeplegmenas*): Aristotle does not explain, either here or elsewhere, what 'compound' predicates are, but *On Interpretation* 5 does make reference to a compound *sentence* (*logos . . . sunthetos*, 17a22).

It is conceivable that this note originated separately from the materials immediately preceding it and found its way here in the editorial process (whatever it may have been, and whoever may have done it) which led to the creation of our present text. It describes in broad terms what Aristotle has been talking about in A 34–36, but without acknowledging that he has been talking about it. There are other cases in the *Prior Analytics* in which a detailed discussion of a subject is followed by a briefer and more primitive treatment of the same thing (see, for instance, B 17 and the Notes on it). It is plausible to ascribe these to editorial practice, though we simply do not know enough about the history of the treatises to determine this with any confidence.

Chapter 38

49a11–12. I translate *epanadiploumenon* as 'something *extra* duplicated' to reflect the prefix *epi* + *ana* (Alexander points out that the middle term is already 'something duplicated in the premises': the case in question involves something extra). (The Latin *reduplicatio* captures this.) Aristotle's subject is deductions in which phrases introduced by 'insofar as' or 'because' are added to a predicate. He classes these generally as 'extra predicates' (*epikatēgoroumena*: cf. 49a25): the *epanadiploumena* are those cases of *epikatēgoroumena* in which the extra predicate is identical to that to which it is added.

The 'reduplicative' statements of this Chapter were the subject of much discussion by later ancient and medieval logicians. For a detailed study, see Bäck *1988*.

49a13–14. Because of the ambiguity of the conjunction *hoti*, *epistēmē hoti agathon* could be rendered either 'knowledge *that* it is good' or 'knowledge *because* it is good.' I have tried to reproduce this ambiguity in English rather lamely with 'knowledge *in that* it is good.' (For a rough parallel, compare

24a22–23: *diapherei . . . hoti*, 'is different . . . *in that*.') In *Posterior Analytics* I.13, Aristotle takes some pains to restrict *hoti* to the sense 'that,' using *dihoti* instead for 'because,' but he does not himself observe this distinction elsewhere (even in the *Posterior Analytics*); it clearly will not fit the present context, since Aristotle varies *hoti* here with *hēi* ('insofar as,' *qua*).

49a18. The expression 'is just a good' (*hoper agathon*), or 'identically a good,' indicates, as Alexander notes, what Aristotle calls 'essential predication': saying of something what it is. This is connected in fundamental ways with the doctrine of 'categories,' or types of predication, elaborated in several places in the treatises: see *Topics* I.9.

49a18–22. The principal point Aristotle makes in this section applies, as he states it, only to first-figure deductions having an 'extra duplication' in their major premises and conclusions. What he says is that in such cases the extra duplication is to be treated as part of the major extreme, not the middle term, since otherwise we get a nonsensical analysis. In effect, he is glossing 'X is Y in that it is Z' as having the structure 'X, in that it is Z, is Y,' rather than the structure 'X is Y-in-that-it-is-Z.' He does not indicate how we should extend the analysis to other cases besides the first figure.

On the phrase 'incorrect and not intelligible' (*pseudos kai ou suneton*), cf. the Notes on 30a27, 37a22, 48a16 ('and' here means 'i.e.').

49a24. 'goat-stag' (*tragelaphos*): this mythical creature is perhaps Aristotle's favorite example of a nonexistent thing (the point being that the term does indeed have a meaning, so that one can, in a sense, know what a goat-stag is, even though there are no goat-stags to have knowledge of).

49a27–b1. In this section, Aristotle evidently has in view a more generalized account of the cases involving 'extra duplication' he has just discussed (although he does not actually make this clear). He now distinguishes deductions 'without qualification' (*haplōs*), i.e., in which the predicate of the conclusion is a simple term with no additional qualifiers, from deductions including such an additional expression. His claim that the 'setting (out) of the terms' is not the same in the two cases again resembles his point in 49a11–26, which was just to explain how to set out the terms in reduplicative cases. (Thus, I take *thesis* in 49a27 as if it were *ekthesis*, and not as 'position.')

What Aristotle says is tantalizingly brief, and most of it is really the treatment of a single pair of examples. Consequently, it is difficult to be certain just what he is up to here, but it appears to be a somewhat sophisticated type of relational deduction. The critical point is determining just what the terms of his examples are. He states the terms of the first deduction as 'knowledge in that it is something' (*epistēmē hoti ti on*), 'being something' (*on ti*), and 'good.' Evidently, the deduction he has in mind is the following:

> There is knowledge of what is so-and-so in that it is so-and-so;
> Being good is being so-and-so;
> Therefore, there is knowledge of what is good in that it is good.

Thus, the expression 'being something' (*on ti*, *ti on*) is a sort of variable here, perhaps best represented as 'being *F*' (see the Note just below on 49a36). Note that here *F* is, in the terms of modern logic, a second-order variable (that is, a variable whose values are predicates, not individuals). The major premise of Aristotle's example is then the second-order statement 'For any *F*, there is knowledge of what is *F* in that it is *F*.' We could then interpret the minor premise as simply the assertion that 'good' is a predicate, so that the entire deduction becomes a matter of instantiating a universal (second-order) statement.

Aristotle contrasts this with the following example:

There is knowledge of what is [so-and-so?] in that it *is*;
What is good *is*;
Therefore, there is knowledge of what is good in that it *is*.

Apparently, 'is' here means 'exists.' (But note that at 43b36–38, Aristotle said that terms which 'follow everything' are useless in the search for deductions.)

If this analysis is on the right track, then what Aristotle is doing in this section is developing a type of generalization of the reduplicative premises considered previously (it is clear that his ideas here are relatively unfinished).

49a28–29. Aristotle lists three types of cases according to the nature of the qualifier attached to the predicate: (1) the qualification is 'this something' (*tode ti*), (2) the qualification is 'in some respect' (*pēi*), (3) the qualification is 'somehow' (*pōs*). He does not tell us what each of these is, and in fact, he may have no very precise system of classification in mind. The expression *tode ti*, literally 'this here something,' is an Aristotelian coinage which usually indicates a particular sensible individual. Alexander (369.33–370.6) takes it in the present case as equivalent to *ti esti* and supposes that in case (1) the qualification must be part of the essence or definition. He illustrates cases (2) and (3) with the examples 'The healthy is knowable insofar as (*qua*) good' and 'A goat-stag is thinkable insofar as it is nonexistent' (*doxaston hēi mē on*).'

49a36. 'for "being so-and-so" was a symbol for its peculiar being' (*to gar ti on tēs idiou sēmeion ousias*): this puzzling expression makes most sense, I believe, as Aristotle's own explanation of his use of 'being so-and-so' (*on ti*, *ti on*) as a kind of variable. Aristotle is, thus, telling us something about the *expression* 'being so-and-so.'

49b1–2. 'It is evident . . . particular deductions': Alexander explains this puzzling remark with the suggestion that 'good in that it is good' is a term of lesser extent than 'good' (so that he takes Aristotle's examples to be in some way particular, not universal, deductions). But Aristotle says, not that these deductions are to be regarded as particular, but rather, that particular deductions are (also?) to be treated in this way.

Chapter 39

The point of this little note is to recommend the substitution of single words as terms in place of their expansions into phrases (e.g., as definitions). 'Have the same value' (*dunatai to auto*): the verb *dunasthai*, 'be able,' is also used with the sense 'be worth' (as in connection with coins), and it occurs as early as Herodotus in the sense 'mean' (of words).

Chapter 40

From a modern viewpoint, the difference between 'Pleasure is *a* good' and 'Pleasure is *the* good' is that the former is a predication, whereas the latter is an identity: the occurrence of the same word 'is' in each is a superficial similarity which conceals a diversity of logical form. Aristotle instead treats them as categoricals but with different predicates ('good' and 'the good' respectively). This tells us that he recognized these as different, although it gives us no clues as to how he understood that difference.

Alexander takes the difference between the two examples to be similar to that between using a term in its normal predicative sense and using that term as the name of a species (to modify his example slightly, the following fallacy illustrates the point: 'Every man is an animal, animal is a genus, therefore every man is a genus'). The distinction between '*A* is *B*' and '*A* is the *B*' attracted Plato's attention also (see, for instance, *Hippias Major* 287a–d).

Chapter 41

49b14–32. The distinction made in this section seems clear enough, but its relationship to the rest of *Prior Analytics* A is less certain. Aristotle's concern is the forms '*A* belongs to all of what *B* belongs to' and '*A* belongs to all of what *B* belongs to all of.' His point is that the former need not be interpreted as synonymous with the latter. Alexander says two things about this section, both of them likely correct. First, he connects the discussion with the *dictum de omni* of 24b28–30: '*A* is predicated of every *B*' means 'none of *B* can be taken of which *A* cannot be said'. Next, he links the discussion to the 'prosleptic' premises which appear briefly in B 5–7 (58a29–32, 58b8–10, 58b37–38, 59a28–29). He also informs us that Theophrastus held the difference between the two forms to be purely verbal, both being equivalent to '*A* is true of all of what *B* is true of all of' (379.9–11).

Alexander takes this last form to be equivalent to, or to imply, '*A* belongs to every *B*.' If we suppose it to be a logical truth that any term is universally true of itself, then the inference from '*A* is universally true of whatever term *B* is universally true of' to '*A* is universally true of term *B*' may seem trivial (because *B* is trivially among the terms *B* is universally true of). For Aristotle, however, the status of such identical predications is at least problematic. He rarely uses them as examples; and although he regards their denials ('Some *A* is not *A*') as obvious falsehoods (see B 15), he never states positively that they are themselves true, and might even have viewed them as in some way ill-formed.

49b30–32. 'And if B is said of all of something . . .': more fully, 'if B is said of all of some third thing, then A is likewise said of all of it; but if B is not said of all of this third thing, then A need not be said of all of it.'

49b33–50a4. The crucial factor in interpreting this section is determining just what Aristotle is talking about. Alexander and Mignucci take him to be defending the use of letters in expounding his deductive theory. On that view, the critical point of comparison is between letters and the actual diagrams used in a geometrical proof: each is, strictly speaking, a 'particular thing' (*tode ti*). Now the geometer may make statements in the course of his proof which he knows to be false about the figure in question (e.g., '*AB* is a straight line one foot long'). If it is the falsehood of these assertions about the actual figure that is important here, then the issue would be how a correct proof could rest on what seem to be false premises. Aristotle, however, does not discuss the falsehood of the premises, but rather, the particularity of the figure: the problem is that this particular figure seems to enter into the proof, which purports to be universal. His response that nothing which is not 'as a whole to a part' (*hōs holon pros meros*) enters into a proof is directed at this latter problem: it is universals, or universal premises, that are related to other things as whole to part, and the actual diagram is not so related to anything. (Ross cites *Sophistical Refutations* 178b36–179a8, where it is clear that the issue involves confusing particulars and universals.)

How then does this apply to the use of letters in Aristotle's own proofs of deductions? To see this, we should remember that his proofs of deductions follow the structure common in Greek mathematical proofs, in which the theorem to be proved is enunciated and recapitulated in general language, normally without the use of letters, while letters are introduced in an intermediate step, at the beginning of the proof proper. (This stage of the proof was called the *ekthesis* by later commentators on Euclid.) Evidently, Aristotle's point is that the *actual letters* which he introduces in the course of the proof are not universals in relationships of predication: they are concrete, sensible individuals, just as the diagrams in geometrical proofs are.

Aristotle may have in mind an even closer connection with geometrical proofs if, as is possible, he used letters as part of some diagramming technique (see Einarson *1936*, Rose *1968*). Unfortunately, we have no idea what it might have been. As I have interpreted it, this discussion has no special connection with the procedure usually called 'proof by *ekthesis*.'

49b32. 'of all': the manuscripts add 'A, B, C' here; Ross follows Alexander's text and the Aldine edition in omitting it.

49b33–34. 'any absurdity results . . . setting something out': the phrase *para to ektithesthai ti sumbainein atopon* is grammatically ambiguous. I take *ti* as the object of *ektithesthai* ('setting something out'); other translators take it to modify 'absurd,' giving a sense like 'an absurdity, should one result, is not to be attributed to the setting-out.'

49b35–36. 'calls this a foot-long line, this a straight line, and says that they are breadthless, though they are not' (*tēn podiaian kai eutheian tēnde kai aplatē einai legei ouk ousas*): commentators disagree over whether 'are' (both participle *ousas* and infinitive *einai*) should be taken as existential or predicative. I have opted for the latter course: Aristotle's example clearly concerns a geometer who says things about a diagram that are not strictly true of it (such that a certain line is really straight or really without breadth). The text here is somewhat difficult grammatically and the number of variations suggests it may be corrupt, although I think the point Aristotle is trying to make comes through well enough. To get the sense I do out of Ross's text, I need to treat the first *tēn* as a demonstrative and take the grammar as somewhat elliptical. Barnes omits the first *tēn* and changes the plural participle *ousas* to the singular *ousan*, giving 'this line is a foot long, and straight, and without breadth, when it is not.'

50a1–2. This sentence (*tōi d'ektithesthai houtō chrōmetha hōsper kai tōi aisthanesthai ton manthanonta legontes*) has vexed translators and commentators. The assumption has been that Aristotle is talking about the use of the *procedure* of *ekthesis* here and comparing it to some use of sensory perception in the case of a 'learner,' i.e., a student. But the grammar of the sentence makes it extremely difficult, if not impossible, to get such a sense out of it. Ross makes the inventive suggestion that we read *ton manthanont' alegontes*, 'in taking care of the learner'; but this requires us to read the unusual poetic word *alegontes*, otherwise not found in Aristotle. My translation is grammatically and textually unproblematic, and leads to a simple connection with the argument of the passage: the *expression* 'set out' no more entails that we are really 'setting out' a concrete individual than the *expression* 'perceive' implies actual sense perception. (Compare the English use of 'see' with the meaning 'understand.')

Chapter 42

50a5–10. 'Deduction' here means an extended deduction which may contain subsidiary deductions in several figures. There is no detailed treatment of such complex deductions in Aristotle, though some results are established in A 25 (see 42b1–26).

Chapter 43

50a11–15. Two things about this isolated note suggest the environment of the *Topics*. First, the subject itself recalls that work's division of arguments as they concern definition, peculiar property, genus, or accident. Second, Aristotle twice uses the verb *dialegesthai* in the sense 'argue (to a conclusion),' a usage ubiquitous in the *Topics* but rare in the *Analytics*.

One striking point about this brief text, which is not evident in the translation, is the number of different senses in which Aristotle manages to use the same two words in it. When he announces his subject as 'arguments aimed towards a definition,' the word *logos* is used in the sense 'argument'; a few lines later the same word means 'definition' ('the entire definition'). When he states his subject, 'definition' is *horismos*; by the end of the sentence, at the phrase 'terms in the definition,' he has switched to the term *horos*. He then proceeds to use the word *horos* in the sense 'term' (which is its usual meaning in the *Prior Analytics*).

Chapter 44

50a16–19. This treatment of 'deductions from an assumption' should be studied in conjunction with the earlier discussions in A 23 (41a21–b1) and A 29 (see the Notes on those passages). Aristotle repeats his view that the conclusions of such arguments are really not 'proved by means of a deduction' (*dia sullogismou dedeigmenoi*) but 'consented to by means of an agreement' (*dia sunthēkēs homologēmenoi*: cf. 41a38–b1). Thus, these arguments are deductive only *from* (*ek*) *an assumption*, i.e., deduce from an assumption as a premise: they are not really *deductions* of their ultimate conclusions, but of something else.

50a19–26. 'for example, if someone assumed': In this example, Aristotle uses the language of disputation and dialectic: when the assumption has been agreed upon, the person deducing 'argues' (*dialechtheiē*) the conclusion: the verb *dialegesthai* implies a context of dispute between two parties (see the Note on 50a11–15). Aristotle explicitly says that the desired conclusion 'has not been proved' (*ou dedeiktai*). It is 'necessary to agree' only in the sense that, otherwise, there would be no argument.

The example Aristotle uses here is alluded to several times, in various ways, in the *Prior Analytics* (see 24b20–21 and Note, 48b4–5 and Note, 48b16–17, 69b9–26). The use Aristotle wants to make of the example seems clear enough: in order to show that there is not always a single science for any given pair of contraries, which has as its object both members of that pair, we first secure an agreement that, if there is not a single potentiality or 'power' (*dunamis*) for a pair of contraries, then there is not a single science of them either. There are certain complications about the example itself, however. The deduction whereby it is shown that there is not a single potentiality for every pair of contraries is apparently itself a deduction through impossibility; thus, on Aristotle's analysis, it too is an 'argument from an assumption.' It is at least curious that he should use so unnecessarily complex a case (since it is, in effect, an argument from an assumption *included within* an argument from an assumption).

50a21–23. 'for example, of what is wholesome and what is unwholesome': the majority of translators take *hugieinon* and *nosōdes* here to mean 'healthy' and 'diseased,' which is certainly possible. However, Aristotle evidently regards it as impossible for there to be a single capacity for these contraries in this case, and that is not, at any rate, an obvious absurdity given his views: in several places he suggests that 'every potentiality is of opposites,' and even if this is restricted to 'rational' capacities as in *Metaphysics* IX.2, it appears to be closely associated with the *Prior Analytics*' notion of two-sided possibility (cf. the Note on 32b4–22). And in any event, there is nothing paradoxical whatsoever about holding that the same person may, at the same time, possess both the potentiality for being healthy and the potentiality for being diseased.

We get a much better sense if we take *hugieinon* and *nosōdes* as 'what is *productive of* health,' 'what is *productive of* disease,' as Alexander does (387.1–5; cf. Pacius, 'salubre et insalubre'). The argument to an impossible conclusion is then as follows: if the potentiality (power) of the wholesome were the same as the potentiality (power) of the unwholesome, then the wholesome would have the power of producing disease. But then the same thing would be at once wholesome and unwholesome, which is absurd. I borrow the felicitous translations 'wholesome,' 'unwholesome' from O. F. Owen's 1853 version.

50a27. 'the latter . . . the former': in Greek, 'the latter' is masculine in gender (*houtos*), while 'the former' (*ekeino*) is neuter. The masculine may be explained by the gender of 'deduction,' which actually *follows* it in the sentence.

50a29–38. The 'leading away to an impossibility' (*hē eis to adunaton apagōgē*) is that part of a proof through impossibility in which the 'impossible' conclusion is deduced.

50a39–b4. This note again indicates the unfinished state of Aristotle's views on 'deductions from an assumption' (cf. 45b12–20). The promised fuller treatment is not found in any work known to us.

Chapter 45

According to its last lines (51b3–5), this Chapter brings Aristotle's discussion of the third main subject of Book A (resolving arguments into the figures) to a close. (A 46 is quite clearly an appendix with no close relationship to the projects of the rest of the Book: see the Notes on it.) However, the contents of A 45 are not really part of the project of A 32–44 (explaining how to 'resolve' given arguments into figured form) but instead address a theoretical problem in Aristotle's deductive system: are completions through impossibility necessary? Aristotle explores this by investigating the ways in which one deduction can be transformed into another by way of premise conversions. The location of this material here may result from the fact that Aristotle also calls this procedure 'resolving' (50b30, 33, etc.). An editor may, therefore, have tacked it on here, even though its more natural place would be with the studies in A 23–26.

50b5–9. The subject of the Chapter is not 'resolving' an arbitrary given argument into a figured argument, but 'leading back' a given *figured* argument from one figure into another. In the case of transformations from other figures into the first, Aristotle's procedures are identical to those proofs he uses in A 4–6 which rely entirely on conversions; here, however, he also investigates all possible transformations from one figure to another. Patzig (pp. 46–47) thinks that Aristotle is investigating alternative axiomatizations of this theory, but the closing remarks in 51a40–b2 are not, in my opinion, congenial to this.

51a24–25. 'when this premise . . .': this remark applies to the resolutions of the first and third figures into one another. 'Replaced' (*metatithemenēs*): compare the use of the closely related verb *metalambanein* at 37b15. Aristotle's claim is true only in something of a Pickwickian sense: none of the universal deductions in the third figure is resolved into the third, while *Disamis* in the third figure must have its premises interchanged (51a9–10: 'B must be put as the first term'), so that it is not really the minor premise which is converted in this case.

51a26–27. 'One . . . the other': since third-figure deductions always have particular conclusions, the only second-figure cases Aristotle even considers for resolution into this figure are the two with particular conclusions (*Festino*, *Baroco*).

Chapter 46

This section is not connected in any close way with the remainder of the *Prior Analytics* and in fact shows more kinship with *On Interpretation*.

51b5–8. The contrast between 'not to be this' and 'to be not this' is expressed in Greek as a matter of the position of the word *ou* (*ouk*), 'not,' with respect to the predicate: in 'is not white' (*ouk esti leukon*) the usual denial of 'is white,' the 'not' is attached to the verb, whereas in 'is not-white' (*estin ou leukon*) it is attached to the adjective. The latter sentence is not quite natural Greek: the hyphenated 'not-white' reflects this. I try to avoid the prefix 'non-' when I can, since this suggests a lexical rather than a syntactical distinction that does not exist in Greek. It should *not* be assumed that 'not white' and 'not-white' in my translation perfectly reflect distinctions of *sense* in Aristotle's text, however.

Aristotle's purpose is to show that 'it is not white,' rather than 'it is not-white,' is the denial of 'it is white.' In the *Topics*, he had treated affirmations and their negations as pairs of *predicates*, comparable to contraries or to conditions and their privations. On such a view, 'not-white' is indeed the negation or denial of 'white.' This view affects his understanding of the principles of excluded middle and noncontradiction, which he often states as 'of everything, either the affirmation or the denial is true' and 'an affirmation and its denial cannot belong to anything at the same time.' But in the present Chapter, he, in effect, argues that affirmations and their denials are pairs of *sentences*, or better (in modern terminology) pairs of *open sentences*: sentences in which the subjects are only variables, such as 'it is white' (where 'it' functions as a variable). Compare his views here with *On Interpretation* 10.

51b10–25. The first argument rests on an analogy: 'is able to walk' is to 'is able to not-walk' (*dunatai ou badizein*) as 'is white' is to 'is not-white.' (I translate *ou badizein* as 'to not walk': see the next Note.) Aristotle then argues that 'is able to not walk' is not the same in meaning as 'is not able to walk,' the denial of 'is able to walk,' since 'is able to walk' and 'is able to not walk' can be true of the same individual at the same time. It follows by the assumed analogy that 'is not-white' is not the same in meaning as 'is not white.' (In the course of the argument, Aristotle switches without explanation to the example 'is not good/is not-good'.)

51b18. 'being-able to not walk': here, we find two very similar phrases meaning 'not to walk' (differing only in the word used for 'not': *ou* in one case, *mē* in the other) joined by 'or' (*ou badizein ē mē badizein*). The phrase 'or not to walk' (*ē mē badizein*) has made this sentence difficult for translators. Alexander's text evidently read 'able to walk or not walk' (*dunamenos badizein ē mē badizein*), omitting the first 'not,' but it is difficult to see how this makes sense. Many translators (e.g., Tredennick, Rolfes, Jenkinson/Barnes) simply omit the second 'or not to walk,' while others (Colli, Tricot, Mignucci) try to translate it as it stands. In fact, Aristotle's point is inseparable from a detail of Greek syntax. There are two Greek words for 'not' which are used in different grammatical contexts: *mē* and *ou*. Among other differences, *mē* is the particle normally used to negate infinitives. Even though Aristotle seems rather casual about substituting *mē* and *ou* for one another in many contexts, his usage is very consistent on this point: he rarely uses *ou* to negate an infinitive, and when he does, it can virtually always be explained by grammatical factors not present here (the context is always governed by a verb of saying or thinking). I believe this permits us to understand what is going on in the present passage.

Aristotle has been comparing 'is not white' (*ouk esti leukon*) with 'is not-white' (*estin ou leukon*). Accordingly, here he considers the parallel relation of 'is not one able to walk' (*ouk esti dunamenos badizein*) to 'is one able to not walk' (*esti dunamenos ou badizein*), simply changing the position of the 'not' as before. However, following this transformation strictly results in the anomalous *ou badizein*. Accordingly, Aristotle adds parenthetically *ē mē badizein*: 'or <as we say it> "not to walk"'. I have tried to reflect the difference by translating *ou badizein* with a split infinitive ('to not walk'), which I would like to think captures about the same degree of ungrammaticality in English.

51b24. 'terms in analogous relationships . . .': the Greek phrase is very brief, but the meaning is clear. The term *analogon* really means 'proportional,' but Aristotle sometimes uses it in a broader sense approaching 'analogous,' normally with a four-term analogy in mind ('A is to B as C is to D'). Expressed more formally, his claim here would be 'if A is to B as C is to D and A is different from B, then C is different from D.'

51b31–35. Aristotle's reasoning in this difficult passage may be reconstructed as follows: we have established that 'is not-good' is not a denial; however, whatever is true of another thing is either an affirmation or a denial; therefore, 'is not-good' is a (sort of) affirmation. I take 'an affirmation or denial' here to indicate a sort of disjunctive predicate, as it sometimes does in Aristotle's expressions of the law of excluded middle (*that which* is necessarily true of anything is the complex composed of the affirmation and the denial joined by 'or'). The phrase 'of every single predicate' is in Greek just 'of every single' (*kata pantos henos*), which is clearly an abbreviation. Alexander (401.25–30) takes it to be elliptical for 'of every single (declarative) sentence' (*kata pantos [apophantikou] logou*), which is not, in fact, greatly different from my own reading. The word 'single' is meant to distinguish simple predicates (or statements, on Alexander's interpretation) from those which are complex, i.e., composed of other constituent predicates (or statements). Compare Aristotle's frequent reference to 'single' statements in *On Interpretation*.

51b36–52a24. This account is, in many ways, closely parallel to *On Interpretation* 10–11, although the latter is much more polished and deals with a wider class of cases. As in *On Interpretation*, Aristotle first discusses unquantified cases (51b36–52a14) and then extends the account to quantified, categorical sentences (52a18–24). The unquantified case appears to concern predication of individuals, not indefinite statements, although this is not clear (it is much clearer in *On Interpretation* 10).

52a1. For no evident reason, Aristotle suddenly changes his example from 'good' to 'white.'

52a15–17. In other words, we may replace 'not-*A*' with the privative of *A*, if it has one (e.g., 'unequal' for 'not-equal,' as here). The term 'predication' (*katēgoria*) is here used to designate the opposite of 'privation' (instead of Aristotle's more usual *hexis*, 'possession').

52a24–38. Aristotle now draws a corollary more closely connected with the subjects of the *Prior Analytics*: since affirmative and negative statements are proved in different ways, 'is not white' and 'is not-white' will have different kinds of proofs.

52a28. 'or that it is possible . . . not to be white': Aristotle is giving us examples of genuine negative statements, to be contrasted with 'is not-white,' which he has just argued is really affirmative. But the second example, 'it is possible for an animal not to be white,' is a statement of possibility, and he earlier said that all such statements are really affirmative. Alexander, therefore, interprets the second example as 'it is possible for *no* animal to be white,' i.e., as a necessary universal negative (411.14–24), but this is very strained. Nothing actually turns on this second example: it is at least possible that Aristotle had not settled on his doctrine that possible statements are affirmative when he wrote this passage.

52a31–32. 'first figure': the word 'figure' here means 'mood' (in this case, *Barbara*). I suggest in Smith *1982a* that Aristotle at one time worked with a 'system' consisting of the four deductive forms *Barbara*, *Celarent*, *Camestres*, *Cesare* (see, for instance, *Posterior Analytics* I.21, 82b15–16, 29–31).

52a34–35. Aristotle's text literally reads 'if it is true to call whatever is a man musical, or not musical'; I have supplied the subject 'the thing to be proved'. Ross, who understands the passage as I do, conjectures that *esti*, 'is,' should be *estai*: 'if it is to be true.' But this emendation is not necessary.

52a38. 'the three ways': that is, the three ways of deducing a negative conclusion (see Note on 52a31–32).

52a39–b13. Aristotle now generalizes the material in 51b36–52a14. The relationship between *A* and *B*, and also between *C* and *D*, is materially equivalent to contradiction: exactly one of the pair must be true of anything. '*A* follows *C* and does not convert with it' means 'Everything *C* is *A*, but not everything *A* is *C*.' What Aristotle proves, stated categorically, is that some *A* is *D*, that no *B* is *C*, that every *B* is *D*, and that not every *D* is *B*. In set-theoretic terms, the pairs <*A*, *B*> and <*C*, *D*> both partition the universe, but *C* is a proper part of *A*; it follows that *B* is a proper part of *D*, that *A* and *D* overlap, and that *B* and *C* are disjoint.

The argument pattern discussed here very closely resembles the argument used in *On the Heavens* I.12, 282a14–22.

52b14–34. The 'failure to take correctly the opposites' here evidently rests on a confusion between 'X is the contradictory of "Y and/or Z"' and 'X is the contradictory of Y and/or of Z.' It is somewhat difficult to follow Aristotle's argument: he, in effect, shows us that a certain line of reasoning leads to an incorrect result and attributes the error to a mistaken principle. (I have tried to make the structure clearer using quotation marks and a few extra phrases; Tricot's translation uses similar devices.)

Aristotle's argument is a sophisticated indirect proof resting on the result just proved in 52a39–b13. Suppose the same situation as in that proof, and take *Z* and *H* to be the contradictories of '*A* or *B*' and '*C* or *D*' respectively. Now make the additional supposition that *Z* is also a contradictory of *A alone*, and likewise *H* of *C* alone. Substituting *Z* and *H* for *B* and *D* in the previous result, we get 'everything *Z* is *H*.' Next, take *Z* and *H* to be contradictories of *B* and *D* respectively: then, since we have just shown that *H* follows *Z*, we may substitute *H*, *D*, *Z*, *B* for *A*, *B*, *C*, *D* in the original theorem, giving the conclusion 'everything *D* is *B*.' But the entire argument rests on nothing beyond the premises of the original theorem and the assumption that the contradictory of a disjunctive term is also a contradictory of each of its disjuncts. Consequently, using that principle, we have proved that, in general, if *A* entails *C*, then the contradictory of *A* entails the contradictory of *C*, which Aristotle knows to be false from the previous proof. Aristotle concludes that this principle is erroneous: the contradictory of '*A* or *B*' is not a contradictory of '*A*' or of '*B*' separately.

Given the way he has set the example up, '*A* or *B*' will apply to everything, and thus 'neither *A* nor *B*' will apply to nothing. Surprisingly, Aristotle takes no note of this.

52b27. 'we know this': i.e., we have just proved it in 52a39–b14.

52b28–29. 'the consequence was in reverse order': see 52a4–12, 52a39–b14 (and compare *Topics* II.7, 113b15–26).

52b33–34. 'the denials which were taken are two': i.e., *Z* and *H*, the 'denials taken,' are each assumed to be *second additional* denials of *A* and *C*, in violation of Aristotle's principle (as defended in *On Interpretation*) that there is a single denial of a single expression.

NOTES TO BOOK B

Chapter 1

52b38–53a3. 'We have already': Ross takes this opening sentence to summarize the contents of the whole of Book A, in three divisions: A 4–26 ('the number of figures . . . of premises'), A 27–31 ('what sorts of things one must look to . . . any discipline whatever'), and A 32–46 ('the route through which we may obtain the principles . . . '). But while the first two divisions correspond to Aristotle's own summary in A 32 (46b38–47a1), the third division planned there is 'how we can lead deductions back into the figures,' which is the real subject of A 32–44; the third subject mentioned here is actually treated in A 27–28 (cf. 43a20–21).

53a2. 'discipline' (*methodos*): Aristotle uses this term frequently to designate both the activity of pursuing scientific inquiry concerning a given subject, and the results of such a pursuit. Etymologically, *methodos* would mean 'pursuit'; except for later Greek, however, it is mostly used in association with the pursuit of knowledge and similar activities. In some cases, Aristotle uses it to include the procedures by which a science is developed (see for instance 46a32 and Note); obviously, our word 'method' is historically connected with this.

53a3–b3. 'Now, seeing that': In this section, Aristotle investigates the conditions under which a deduction 'deduces several results' (*pleiō sullogizetai*). He considers two different sorts of cases of this, and it is not fully clear what they have in common. In 53a3–14, he shows that an additional conclusion can be derived from many deductive forms through conversions. By contrast, in 53a15–b3 he is concerned with conclusions which are deduced from an original deduction with the addition of another premise. It is difficult to see how these claims are related to any other projects of the *Prior Analytics*, or how, exactly, they are related to one another. Conceivably, he is trying to explore how a collection of deductions fits together in the structure of an entire demonstrative science. This is a subject about which he has comparatively little to say in either the *Prior* or the *Posterior Analytics*, although he does recognize its importance. If that is his concern, then it may be relevant to compare this passage with A 25.

53a3–14. These results are a simple consequence of the conversion rules of A 3: additional conclusions can be derived from all deductions save those with *o* conclusions. As commentators observe, this implies that Aristotle was aware of all the so-called 'subaltern' moods, together with the remaining fourth-figure moods not already included in A 7, 29a19–29.

53a7–8. 'privative <particular>': Aristotle says only 'privative,' but he must have *o* premises in mind.

53a12. 'this conclusion is different from the previous one': i.e., 'B to no A' is a different conclusion from 'A to no B.' Although this seems too obvious to require mention, Aristotle occasionally speaks as if the converse of a universal negative is the same as it (e.g., B 5, 57a27–29). Since in Book A Aristotle clearly takes them to be different, these may be traces of an earlier position; against such a background, the present sentence would have more point.

53a15–b3. 'This cause': Aristotle now proceeds to give 'another account' of 'deducing several results': literally, to 'discuss them differently' (*allōs eipein*). What follows is not (as some commentators suppose) an alternative explanation of the result he has just proved, but a discussion of a different type of *pleiō sullogizesthai*.

Aristotle tells us that in universal deductions it is possible to prove a variety of conclusions *in the same deduction* by considering various *other* terms which fall under the middle or minor term. (He uses the word 'conclusion,' *sumperasma*, to mean 'minor term'; I translate it here as 'conclusion-term.') Thus, to take his first example, suppose we have the deduction '*A* belongs to every *C*; *C* belongs to every *B*; therefore, *A* belongs to every *B*.' This deduction shows that '*A* belongs to whatever is below *B* or *C*': if we then take some term *D* which is below *B*, we can deduce that *A* belongs to every *D*, etc. In a similar way, he argues that in a universal negative deduction '*A* belongs to no *C*; *C* belongs to every *B*; therefore, *A* belongs to no *B*,' we can deduce additional conclusions *in the same deduction* by taking a term below *B* or *C*. Taken at face value, this implies that all first-figure deductions with the same major premise are the same. It is difficult to understand how Aristotle could call this the 'same deduction' after taking the care he does in A 25 to argue that every *single* deduction is through exactly three terms: surely what we have here is just *another* deduction, with different terms.

There are further difficulties. First, in dealing with the second figure, Aristotle only discusses the case of *Cesare*, with the major premise negative; Waitz notes that his argument cannot be applied to *Camestres*. In addition, as Ross points out, the closing lines (53b1–3) seem to retract the entire point of the section.

It is difficult to be sure what the two cases of 'deducing several' have in common, but one possibility is this: in each case, the same relationship of terms is at least *involved* in the deduction of more than one conclusion. Aristotle tends to identify deductions with their premises, and premises with their terms. Therefore, he may be investigating the question how terms in a given relationship can play a role in more than one deduction. This would be an important part of a study of the entire structures of demonstrative sciences, in which the same premise might appear in several deductions (as in Aristotle's second case), or the same premises might yield conclusions used as premises in different further deductions (perhaps related to Aristotle's first case). Some of the details noted above suggest that Aristotle had not yet worked out his theory of deduction when he wrote these lines.

53a32–33. 'taken as undemonstrated': in the *Posterior Analytics* the term *anapodeiktos* usually means 'not demonstrable,' that is, 'not susceptible of demonstration.' Here, however, it means only 'not deduced from premises' (compare 58a2).

53a40. The deduction 'already formed' (*progegenēmenos*) is the initial deduction '*A* to every *B*, *B* to some *C*.'

53a40–b3. The reference to 'the other figures' is puzzling. Since he has already discussed universal deductions in both first and second figures but only mentioned first-figure particular deductions, it seems most likely that he means 'the same thing goes for particular deductions in the other figures.' But in the second figure, it holds, at best, only for deductions with a negative major premise, which can be converted (and thus not for *Camestres* or *Baroco*); and in the third figure, it does not seem to hold for deductions with particular major premises (*Disamis*, *Bocardo*).

Chapter 2

Chapters 2–4 form the first of a series of systematic investigations of properties of deductions in all the figures which continues through B 15.

53b4–10. 'Now, it is possible': The general point which Aristotle wants to make in B 2–4 seems straightforward enough: a deduction with true premises cannot have a false conclusion, but it is possible for a deduction with one or more false premises to have a true conclusion. The importance of this result, however, is greater than at first appears. *Posterior Analytics* I.12, 78a6–13, indicate that some of Aristotle's contemporaries probably assumed that a deduction could be discovered by a process of 'analysis,' which amounted to a sort of attempted deduction of the *premises* from the desired *conclusion*. Showing that one cannot infer the falsehood of the conclusion from the falsehood of the premises, or the truth of the premises from the truth of the conclusion, counts decisively against this program.

53b8–10. 'except that': the mention of the distinction between a deduction of the 'why' (*to dihoti*) and a deduction of the 'that' (*to hoti*) is somewhat puzzling here. According to *Posterior Analytics* I.13, a deduction of the 'why' is a full-fledged proof, or demonstration, in which the premises are not only true, but also give the reason or explanation for the conclusion's truth. A deduction of the 'that' is presumably a deduction which fails to be a proof in this sense. Interpreted broadly, this could even include deductions with false premises, and the commentators generally take it so. However, the discussion in the *Posterior Analytics* only concerns deductions with true premises. Aristotle tells us that 'The planets are near; what is near does not twinkle; therefore, the planets do not twinkle' is a demonstration, whereas 'The planets do not twinkle; what does not twinkle is near; therefore, the planets are near' is only a deduction of the 'that,' even though each has true premises: the second, unlike the first, fails to give the cause of its conclusion. (See the Note below on 65a10–25.)

53b10. 'will be explained': this promise is ambiguous, since it might refer either to the claim that a deduction of the 'why' from falsehoods is not possible, or to the general point of B 2–4 that, although a false conclusion may not follow from true premises, a true conclusion may follow from false premises. Commentators have universally taken it in the first way, supposing the reference to be to the concluding lines of the entire discussion (B 4, 57a36–b17). Patzig, while agreeing that the promised explanation is to concern deductions of the 'why' and the 'that,' nevertheless argues in detail that 57a36–b17 is not concerned with that issue at all, and takes the reference to be, instead, to *Posterior Analytics* I.2, wherein Aristotle includes among the conditions a deduction must satisfy in order to be a demonstration the requirement that its premises be true (Patzig *1959*).

Patzig's argument that 57a36–b17 is not an explanation why a deduction of the 'why' cannot have false premises is persuasive, but his identification of the reference is less satisfactory. Aristotle does not *explain the reason* that a deduction of the why cannot be from false premises, either in *Posterior Analytics* I.2, nor anywhere else in the *Posterior Analytics*: this is rather a matter of definition. (If an explanation were wanted, it is not clear what it would need to be: it is like trying to explain why, if the sun makes things hot when it shines on them, that the sun is not the cause of things becoming hot when it is not shining on them.) It is more natural to take the promised explanation as simply the entire discussion in B 2–4 (cf. A 45, 50b9, where 'in what follows' simply refers to the remainder of the Chapter), though 57a36–b17 is also appropriate (see the discussion of that passage below).

53b11–15. 'First, then': taking 53b10 as I prefer, we are given at once half the promised explanation: a false conclusion may not follow from true premises because that would violate the law of noncontradiction.

53b16–25. 'But let it not': this passage is a close parallel to A 15, 34a16–24: having just given a similar argument about inferences using the form 'it is necessary for B to be when A is' (*tou A ontos anankē to B einai*), Aristotle takes care to explain that 'A' cannot be a 'single thing,' i.e., a single statement or premise.

53b26–54a2. 'It is possible': in this first section, Aristotle shows that both premises of any first-figure universal deduction can be false in either sense with a true conclusion.

54a2–b16. 'But if only': Aristotle now treats first-figure cases with one true and one false premise: a wholly false major and a true minor (54a2–18), a partly false major and a true minor (54a18–28), a true major and a wholly false minor (54a28–b2), and a true major and a partly false minor (54b2–16). He gives an explicit definition of 'wholly false' (54a4–6); he clearly means that a universal sentence is 'false in part' if its contradictory is true *and its contrary* *false*, though he does not explicitly say so. If we replace the major premise of a first-figure universal deduction with its contrary, we get the premises of another first-figure deduction having a conclusion inconsistent with the conclusion of the original deduction: for this reason, in the first figure we cannot have the major wholly false and the minor true with a true conclusion. All other combinations are possible, however.

54a8. 'AB as wholly false': here and a few lines later (12–13), Aristotle's language shows another type of abbreviation. In 8, we actually find ***tēn de to AB pseudē holēn***: the first article is feminine, the second neuter. Similarly, in 12–13, we have ***hē men to BG alēthēs protasis, hē de to AB pseudēs holē.*** Ross explains the feminine article as an ellipsis for *hē protasis eph' hēi keitai* ('the premise to which applies') and notes the similar construction in a geometrical example at *Posterior Analytics* 94a31.

54a9–10. 'did not belong' : i.e., 'as we established earlier' (in this case, in A 4).

54a11. 'And similarly': As Ross points out, Aristotle begins the long sentence 54a11–15 intending to end it with 'the conclusion cannot be true,' but actually ends it with 'will be false': this makes the 'neither' (*oud'*) in 54a11 out of place. I omit it in translation.

54a29–30. Aristotle refers to a deduction with a true conclusion as a 'true deduction' even in this case, when one of its premises is false.

54a31–32. 'such species of the same genus as are not under one another': this is the first of a number of examples (continuing through B 3) which Aristotle characterizes by means of the vocabulary of the 'predicables,' as they later came to be called. These are relations which two universals, or classes, may have to one another. In the *Topics*, Aristotle officially distinguishes four: definition (*horos*), genus (*genos*), 'accident' (*sumbebēkos*), and 'property' or 'peculiarity' (*idion*). However, he also makes use of a fifth in that work, the 'difference' (*diaphora*). The term 'species' (*eidos*) in Aristotle sometimes means 'lowest species,' i.e., narrowest universal containing an individual (in this sense, an account of an individual's species would be its definition). Here, Aristotle tends, instead, to treat genus and species simply as universals of greater and lesser generality respectively: if A is a universal contained by B, then A is a species of B and B a genus of A. On such a view, 'genus' and 'species' are relative terms, and the same universal can be both a species and a genus.

54b9. 'which was true' (*hoper ēn alēthes*): that is, this conclusion was the one we initially wanted to show could be true without its premises true. Aristotle uses this way of speaking when (as here) he presents the desired true conclusion as the conclusion of a deduction, even though it is known that not all its premises are true.

54b13–14. The terms 'wisdom' (*phronēsis*) and 'theoretical science' (*theōrētikē*: *epistēmē* is understood) are the two principal types of intellectual activity recognized by Aristotle (as in *Nicomachean Ethics* VI). A widely established translation for *phronēsis* is 'practical wisdom' (these are, of course, only examples here).

54b17–55b2. 'In the case of particular': the same results are shown to hold for particular deductions, except that it is also possible for the major premise to be wholly false and the minor true (the reason is that the conclusions of *Darii* and *Ferio* in the same terms are not inconsistent with one another). In order, the cases are wholly false major and true minor (54b17–35), partly false major and true minor (54b35–55a4), true major and false minor (55a4–19), partly false major and false minor (55a19–28), and wholly false major and false minor (55a28–b2). The 'particular' is of course the minor premise.

Chapter 3

Aristotle finds every possible combination of truth values of the premises for second-figure deductions consistent with a true conclusion: both wholly false (universals, 55b10–16; particulars, 56a32–b3), one wholly false and one true (universals, 55b16–23; particulars, 56a5–18); one partly false and one true (universals, 55b23–38); both partly false (universals, 55b38–56a4); or the universal true and the particular premise false (56a18–32).

55b7–9. Ross (432–433) notes a number of reasons why the bracketed parts of this summary do not seem to describe accurately what Aristotle actually does. I follow him in rejecting these words as a later gloss.

55b15–16. 'the same deduction': if the premises of a deduction in second-figure *Camestres* are wholly false, then the premises of a corresponding deduction in *Cesare* are true, and conversely: therefore, for each of these, if the premises are wholly false, the conclusion not only *may* be true, but *must* be. What Aristotle means by 'the same deduction' is not clear: he may mean 'when the premises are wholly false we will have a deduction with the same conclusion.'

55b30. 'when the privative is put in the other position' (*metatithemenou de tou sterētikou*): i.e., with a negative minor rather than major premise. The same use of *metatithenai* follows at 56a4; compare the different sense at 51a24–25.

56a14–15. 'to something white': as Ross points out, this must be taken in the strict sense 'to something but not to everything.'

56a35–36. 'and to some C': as in 56a14–15, this must again mean 'to some but not every.'

Chapter 4

In the first part of Chapter 4 (56b4–57a35) all possible combinations of premise truth-values are again shown to admit a true conclusion: both wholly false (56b9–20), both partly false (56b20–33), one true and one wholly false (56b33–57a9), one true and one partly false (57a9–28). His detailed discussion concerns only the two 'universal' deductions, i.e., those with two universal premises (*Darapti* and *Felapton*): all the particulars are treated in summary fashion (57a29–35).

56b7–8. 'or the reverse': that is, reversing the truth-value assignments to the premises in *each* of the situations just mentioned (true/false, true/partly false).

57a1. After 'the terms . . . are the same,' the manuscripts add the term triplet 'black, swan, inanimate,' which is certainly not the same set of terms as in the preceding example (animal, white, swan). Nor, indeed, could they be: the case needed is third-figure *Felapton* with true major and wholly false minor, so that both extremes need to be universally false of the middle. The trio 'black, swan, inanimate' will work for this purpose if 'swan' is the middle term, since no swan is either black or inanimate and some inanimate things are not black. However, as Ross points out, Aristotle's standard order in stating term-triplets for the third figure (as in A 6) is major-minor-middle, not major-middle-minor as we seem to have here.

There is a similarity between this sentence and 57a8–9, where, because of the symmetry of the premises, the same terms really will serve for two cases (by interchanging major and minor): if the text is not corrupt, then Aristotle has been guilty of a rare oversight.

57a17. 'when the same terms are transposed' (*metatethentōn*): that is, using the same example, but with major and minor terms interchanged. This is still another use of the versatile *metatithenai*: see the Note on 55b30.

57a23–25. 'has been proved': this seems to correspond to the deduction *Felapton*, but it was proved in A 6, 28a26–30, that under the given circumstances, A not only *may*, but *must* not belong to some B. Aristotle may, instead, be referring to 57a1–5, where he gives an example with the required truth values and such that A belongs to some B: if we take this as intended as a strictly particular conclusion ('to some and not to others'), the example might serve.

57a29–35. 'And it is also': Aristotle covers all 'particular' deductions (deductions with one particular premise) with a blanket assertion that the examples used for universal cases can also be used for these. His point depends on the following fact about the third figure: each of the four deductions with a particular premise can be obtained from a universal deduction by replacing one of its premises with the corresponding particular premise (we thus get *Disamis* and *Datisi* from *Darapti*, *Bocardo* and *Ferison* from *Felapton*). Call the two-universal deduction which corresponds in this way to a deduction with one particular premise its *corresponding* deduction, and call the universal premise which corresponds to its particular premise the *corresponding* premise. Then, any model for a corresponding deduction which makes the corresponding premise wholly false makes the particular premise of the deduction false, and any model which makes the corresponding premise false in part makes the particular premise true. Mignucci works through all the details of the correspondence (603–608).

57a36–b17. 'It is evident': Aristotle closes his discussion with a final set of remarks to clarify why a deduction may have a true conclusion and false premises, but not a false conclusion and true premises. The point he addresses is quite precise: even though the conclusion of a deduction with false premises may be true, nevertheless it is not so 'of necessity.' Aristotle supports this with an ingenious argument intended to establish that if A entails B, then it cannot be the case that the denial of A also entails B. (For a detailed study of the passage, see Patzig *1959*.)

What does Aristotle mean by saying that even if the conclusion of a deduction with false premises is true, it is not true 'of necessity'? Obviously, he does not mean 'not necessarily true,' since that would not differentiate such deductions from many with true but nonnecessary premises. A number of commentators take him to mean that the conclusion does not *necessarily* follow in such a case, but Patzig makes clear the hopelessness of this view. Instead, Aristotle surely means the sort of necessity he attaches to any conclusion of any deduction, i.e., 'necessity when certain things are so' or 'conditional necessity' (*tinōn ontōn anankē*); he may also have in mind that the conclusion is not *explained* by the premises in such a case.

Part of the difficulty with understanding this passage is that it is not clear what is wanted here by way of explanation. At one level, it seems sufficient to say that the definition of 'deduction' rules out a deduction with true premises and a false conclusion but not a true conclusion and false premises. This does not, of course, show that cases of the latter sort are in fact possible, but Aristotle's examples do. However, one possible reason for seeking a further explanation might be Aristotle's concern, not simply with deductions in general, but with causal or explanatory deductions (demonstrations).

57b4–17. 'But it is impossible': Aristotle's argument is an intended argument through impossibility using a comparatively sophisticated technique of formal substitution (for a similar case, see *Posterior Analytics* I.3, 72b36–73a6). He first supposes, as a *reductio* hypothesis, that the following are both true:

(1) If A is white, then B is large.
(2) If A is not white, then B is large.

He next says that from premises of the forms

(3) If A is white, then B is large.
(4) If B is large, then C is not white.

we can deduce a conclusion of the form

(5) If A is white, then C is not white.

Next, he states the general principle

(6) If X necessarily is when Y is, then Y necessarily is not when X is not.

Applying (6) to (1), we get

(7) If B is not large, then A is not white.

But this, together with (2) and the inference pattern (3)–(5), gives

(8) If B is not large, then B is large.

Aristotle rejects (8) as impossible, and thus he believes he has deduced an impossibility from the pair (1) and (2).

Some commentators have suggested that the 'impossibility' deduced is not really impossible at all. Łukasiewicz (*1957*, 49–51) pointed out that as a thesis of propositional logic, thesis (8) ('If not-p then p') is simply equivalent to p, and thus proving that a proposition follows from its own denial is proving that very proposition. Moreover, Aristotle himself knows of, and even uses, arguments having this form. Vailati *1911* notes that Plato's argument against Protagoras' 'man the measure' doctrine in the *Theaetetus* and the proof of Euclid IX.12 have this structure; Mignucci points out that an argument in Aristotle's own *Protrepticus* (fr. 2) does also (614–615).

It is of course conceivable that Aristotle might in one place argue in accordance with a principle and (through inadvertence, inconsistency, or a change of mind) explicitly deny that same principle in another place. However, there is an important difference here. It is not as clear that (1) and (2) are jointly possible in application to *explanations*. We may express this difference by substituting 'since' for 'if' in (1) and (2):

(1*) Since A is white, B is large.
(2*) Since A is not white, B is large.

If we interpret 'Since p, q' as 'If p then q, and p,' then obviously (1*) and (2*) cannot simultaneously be true (since that would entail 'A is white and A is not white'). Now, Aristotle clearly does not have this sort of argument in mind. However, we might take (1*) and (2*) as assertions about different occasions: on one occasion the reason why B is large is that A is white, while on another the reason is that A is not white. Now, (6) may be regarded not just as a rule of inference, but as a principle of explanation: if p explains q, then q being false explains p being false. It would follow from this and (1*) that B not being large explains A not being white. The inference (3)—(5) can be given a similar reading (if p explains q and q explains r, then p explains r). If this is what Aristotle has in mind, then (8) does become absurd, since it asserts that the reason why B is large is that it is not large.

57b10. 'the first': the text reads 'A' (i.e., the letter alpha), which as Ross notes can be interpreted as the numeral 'one.'
57b17. 'just as if by means of three terms': the deduction (3)—(5) involves three terms. We get an argument of the same form containing only two terms by substituting *A* for *B* and *B* for both *A* and *C*.

Chapter 5

The subject of Chapters 5–7, 'proving in a circle' or 'proving from one another,' is closely connected with the argument in *Posterior Analytics* I.3 against the possibility of a 'circular' proof of the principles of a science from one another in some fashion (there appears to be an explicit reference to these chapters at 73a6–20). In the *Posterior Analytics*, Aristotle discusses a chain of deductions in which *p* is deduced from *q*, *q* from *r*, and finally *r* from *p*. The position which he attacks was, evidently, that in such a 'circular' case *all* of *p*, *q*, and *r* were proved in virtue of their being deduced from each other. Aristotle's objection is threefold: (1) invoking his requirement that the premises of a proof be epistemically prior to the conclusion, we would get the absurd result that a proposition is prior to itself; (2) such a procedure amounts to deducing a proposition from itself, and thus, anything whatever would admit of this sort of 'proof'; (3) in any event, the procedure can only be applied to a narrow and unimportant class of cases. It is in reference to (3) that Aristotle appeals to the present discussion in *Posterior Analytics* I.3.

The exact relationship of 'circular' proof as defined here to the circular argumentation of *Posterior Analytics* I.3 is not as straightforward as at first appears: in B 5–7, the procedure is not a matter of deducing propositions from one another in a circle, but rather of constructing a deduction of one of the premises of a deduction from the conclusion and the *converse* of the other premise. But with some speculation, we may imagine how Aristotle might have been led from one case to the other. The simplest case of a circular deduction would be two propositions which can be deduced from one another. But although Aristotle implicitly recognizes such cases in the conversion rules for *i* and *e* statements, he refuses to call these deductions (see the Notes on 40b35–36). And, in any event, obviously no circularly proved set of principles for a science can be constructed on this model. Let us, therefore, suppose that every deduction has (at least) two premises. But since no deduction is possible in which *both* the premises can, in turn, be deduced from the conclusion, the next possibility would be a deduction either premise of which could be deduced from the conclusion and the other premise. However, an investigation of all deductions in the figures shows that this can never happen. If we suppose him to have gotten to this point, Aristotle may then have asked: is anything *close* to this situation possible? A simple modification is replacing 'and the other premise' by 'and *the converse of* the other premise': and here we find that under some circumstances such circular deductions are possible. (For a fuller discussion of this interpretation, see Smith *1986*.)

Apart from these formal questions about proof theory, something like deduction in a circle plays a role in Aristotle's conception of change in the sublunary world, where processes often follow a circular parth: see, for instance, *Posterior Analytics* II.12, 95b38–96a7, and *On Generation and Corruption* II.10–11.

There is no suggestion that Aristotle thinks of circular deduction as some sort of transformation which preserves validity, like those studied in B 11–13, so that one could use it to get new deductions from old. What Aristotle actually does is investigate each deduction in the figures, determining when a circular deduction works.

57b18–21. 'Proving in a circle': the *kai* in the phrase *to kuklōi kai ex allēlōn deiknusthai* is epexegetical: *to kuklōi deiknusthai* and *to ex allēlōn deiknusthai* are different names for the same process. (Compare *Posterior Analytics* I.3, 72b17–18: *endechesthai gar kuklōi ginesthai tēn apodeixin kai ex allēlōn.*) The definition would apply to a single deduction of a premise from the conclusion and the converse of the remaining premise, but it seems more likely that by a circular proof, he means an extended structure in which every premise also appears as a conclusion. Such a structure must contain six statements and six deductions. Let p, $q \vdash r$ be the original deduction and let 'conv(p)' denote the converse of p (note that conv(conv$p$)) = p). The full-fledged circular proof structure will then be:

(1) $p, q \vdash r$
(2) r, conv(p) $\vdash q$
(3) q, conv(r) $\vdash$ conv(p)
(4) conv(p), conv(q) $\vdash$ conv(r)
(5) conv(r), $p \vdash$ conv(q)
(6) conv(q), $r \vdash p$

Application of the transformation to (6) then gives us (1) again.

Aristotle refers to circular deduction here in a way that indicates that the expression (or expressions: see above) were in current use in his time, perhaps in the Academy. *Posterior Analytics* I.3 makes it clear that there were partisans of some sort of circular deduction as a method of proving everything, including the first principles themseleves (we do not know who they were: for conjectures, see Smith *1986*).

57b25. Ross brackets 'because' (*hoti*) here, supposing it to have the meaning 'that' and finding it ungrammatical.

57b32–35. Terms 'convert' if they are universally true of each other (or, in the case of a set of three terms, pairwise convertible). Note that if *A* and *B* are convertible terms in this sense, then the sentence *AaB* is also 'convertible' in the sense that it and its converse *BaA* are both true. (Of course, it is not convertible in Aristotle's ususal sense.)

57b33. 'undemonstrated' (*anapodeiktos*): see the Note above on 53a32–33.

57b35–58a12. 'But in the case': Aristotle considers a deduction in *Barbara* and proves that such a deduction can be built up into a complete circular deduction, in which every premise is also a conclusion, if and only if its terms all convert with one another. The strategy of the proof appears unnecessarily complex but is not. Aristotle begins with a deduction

(1) AaB, $BaC \vdash AaC$.

He also assumes that a circular deduction of each premise is possible:

(2) AaC, $CaB \vdash AaB$
(3) BaA, $AaC \vdash BaC$

Now, both the premises of the original deduction have occurred as conclusions. However, two new premises have been introduced: *CaB* in (2) and *BaA* in (3). Applying the circular-proof transformation to (3) and (2) respectively gives:

(4) BaC, $CaA \vdash BaA$
(5) CaA, $AaB \vdash CaB$

At this point, every premise occurring in any of these deductions has also occurred as a conclusion, with the single exception of *CaA* in (4) and (5). An appropriate circular transformation of either (4) or (5) yields the needed result:

(6) CaB, $BaA \vdash CaA$.

58a15–20. 'And it also results:' it is a matter of the definition of circular proof that the conclusion is used in proving each of its premises, but Aristotle probably makes this remark here in light of the results he is aiming at in *Posterior Analytics* I.3 (cf. 73a4–6).

58a27–29. 'the same premise': this remark sheds some light both on Aristotle's concept of a statement or proposition and on the conversion rules: if an *e* premise is *the same premise* as its converse, then conversion inferences are not deductions because nothing *different* follows in them. But this is not always Aristotle's view: cf. 58b25–27, 59a10–14, and the Note on 53a12. The point of the remark is also obscure: the reason there is no deduction when the circular transformation is applied is that both premises are negative. Aristotle may mean that even after conversion, *AeB* is *still* negative, so that the problem of two negatives is not alleviated.

58a29–30. 'Instead, one must take': this 'premise' is the first example of what later commentators designated a 'prosleptic' premise (relying on 58b9: see the Note below). Their general form is: '*A* belongs to all/to none/to some/not to all/ of what *B* belongs to all/to none/to some/not to all of.' Sixteen such forms are possible; though some are equivalent to categoricals (e.g., '*A* belongs to all of what *B* belongs to all of' is equivalent to *AaB*), most are not. Aristotle arrives at the form in an obvious way. The circular transformation applied to *AeB*, *BaC* ⊢ *AeC* gives the two negative premises *BeA*, *AeC*, from which nothing follows. Aristotle therefore asks: what else would it take to get *BaC* from these? The 'prosleptic' premise is simply *constructed* as exactly what we must assume about *B* and *A* to get the desired conclusion. Aristotle never develops a theory of such statements, but recent writers have: see Lejewski *1961*, Kneale *1975*.

58a36–b12. 'In the case': in discussing particular deductions, Aristotle appeals to the rule that a universal conclusion must have universal premises to rule out circular deductions of their universal premises. But he has just allowed the deduction of the affirmative premise of a negative deduction; and in the present case the required 'prosleptic' premise can readily be constructed ('What belongs to some of *C* belongs to all of *B*').

58b1. 'both premises become particular': but in this case the relevant premises (i.e., the conclusion and the particular premise) are already particular, before either is converted. By 'conversion' Aristotle may just mean (loosely) the entire process of converting a premise and trying to use it with the original conclusion to construct a deduction.

58b9. Between 'universal deductions' and 'that is,' some manuscripts add the phrase 'cannot be, but it can be by means of an additional assumption' (*ouk esti, dia proslēpseōs d'estin*). Since this has limited authority and seems not to make sense, Ross rightly condemns it. It is the only place in which the term *proslēpsis* ('additional assumption') is used of one of Aristotle's noncategorical premises: thus, the designation 'prosleptic argument' seems not to be Aristotelian. And even if the passage is Aristotle's, *proslēpsis* has no special connection with these premises: the verb *proslambanein* is frequently used elsewhere of all sorts of 'additional assumptions,' including the additional steps required to complete an incomplete deduction (cf. 58b25–27, 59a11–13, 61a20, 61b7).

Chapter 6

58b15–18. 'the positive cannot be': this reluctance to get an affirmative conclusion from negative premises is not reflected in 58a29–32 above.

58b21. Ross brackets 'and to no C,' though the phrase has reasonably good authority, on the grounds that *BeC*, the conclusion of the original deduction, is *already* assumed. But it is Aristotle's usual practice to mention both premises.

58b25–27. The 'premise taken in addition' (*proslēphtheisēs*) apparently is the converse of the desired conclusion (*BeA*), from which the conclusion follows at once. The use of *proslambanesthai* here shows that the term is not at all associated with noncategorical premises (cf. 61a20, 61b7, and the Note on 58b9).

58b27–29. 'the same reason . . stated previously': that is, at 58a38–b2.

58b35–38. 'for it results': Aristotle's remark here about two negatives would also seem to apply to the first-figure case (*Celarent*), despite the treatment in 58a26–32.

Chapter 7

58b39–59a3. 'when both the premises are taken as universal': Aristotle again rejects getting a universal with a particular premise.

59a8–14. 'For let A belong': this passage is striking in several respects. Aristotle begins with the deduction *AaC*, *BiC* ⊢ *AiB* and, converting the major premise, gets the premises *CaA*, *AiB* for a circular deduction. *CiB*, the converse of the original minor premise, then follows through *Darii*: but Aristotle, taking care to distinguish an *i* sentence from its converse, says that *BiC* has *not* been proved, even though it necessarily follows (which apparently contradicts the definition of 'deduction' at 24b18–20). He then says that it must be 'supposed in addition' (*proslēpteon*) that 'if this belongs to some of that, then that other also belongs to some of this' in order to get the conclusion, and that as a result, the circular deduction no longer rests only on *CaA*, *AiB*. Most striking is the 'additional assumption' of a basic conversion rule: this would describe what happens in most of the completions in A 4–7.

59a32–41. It is difficult to make sense of the bracketed passage on several points, and Ross accordingly rejects it. The greatest difficulty is the statement that, in the first figure, circular proof with a negative deduction comes about through the third figure. Evidently, this is meant to apply only to the 'prosleptic' proof of the minor premise of *Celarent*: Ross makes the plausible suggestion that it is based on a superficial similarity between 'what this belongs to none of the other belongs to all of' and third-figure deductions. The reference to incompleteness is also surprising, since this is the only mention of that concept outside A 1–22.

Chapter 8

59b1–11. Converting as defined here is a transformation performed on existing deductions (cf. 61a21–25 below). 'Replacing' (*metatithenai*) the conclusion is, as we immediately learn (59b6–11), 'converting' it, which means substituting either its contrary or its contradictory for it. (Compare 51a24 and the use of the related verb *metalambanein* at 37b15, 40a34, 56b8.) Aristotle's procedure here is sometimes seen as the derivation, from a deduction *p*, *q* ⊢ *r*, of another deduction *p*, not(*r*) ⊢ not(*q*) or not(*r*), *q* ⊢ not(*p*) (see Patzig *1968*: 152–154). The justification offered in 59b3–5 does appear to recognize the logical validity of such a process; however, Aristotle includes among the pairs of 'contraries' corresponding *i* and *o* statements, which of course are not inconsistent with each other. (At A 15, 63b27–28, Aristotle notes that the 'opposition' of these is merely verbal.) Moreover, the actual procedure of B 8–10 never appeals to this justification. Instead, Aristotle first performs the transformation, taking the 'converse' of the conclusion and one of the premises, and then notes whether these new premises yield the 'converse' of the remaining original premise. The summary at the end of the account (61a5–16) suggests that he is not *appealing* to a rule for deriving deductions but rather attempting to *establish* one by investigating all possible cases. It is worth noting the similarity between 'conversion' and circular proof: in each case, we take an existing deduction and try to get from it a deduction of one of its premises (perhaps transformed) from the conclusion and the other premise (each perhaps transformed). However, in circular proof it was the premise, not the conclusion, which was 'converted,' and 'converted' had a different meaning.

Aristotle gives no explanation of why this procedure of 'conversion' is important. I would speculate that it is dialectical in origin: to convert an opponent's argument is to 'turn it around,' rejecting its conclusion, and thereby rejecting one of its premises (compare the similar definition of converting in *Topics* VIII.14, 163a32–36).

59b11–20. 'For let A': Aristotle's treatment of first-figure *Barbara* exemplifies his approach to all cases. Beginning with a deduction *AaB*, *BaC* ⊢ *AaC*, he first pairs the contrary *AeC* of the conclusion with each premise in turn. The pair *AaB*, *AeC* are the premises of a second-figure deduction which yields *BeC*, the contrary of the original premise *BaC*. However, the pair *AeC*, *BaC* are third-figure premises which yield only *AoB*, the contradictory of the original major premise *AaB*. As the first case makes clear, Aristotle is using his knowledge of deductions in the figures to make inferences, not relying on a logical rule of the sort Patzig supposes.

59b39–60a1. 'conclusion that falls short': i.e., is only the contradictory (not the contrary) of the other premise.

Chapter 10

60b15–18. 'either it is necessary . . . conversion': here, 'conversion' means what it does in A 4–7. In completing deductions, Aristotle always tries first to convert so as to get first-figure premises. The deduction considered here is *Darapti*: *AaC*, *BaC* ⊢ *AiB*. 'Contrary' conversion gives either *AoB*, *AaC* or *AoB*, *BaC*. In the first case, the premises are in the second figure. In the completions of A 5–6, Aristotle always tries to convert second- and third-figure premises so as to get first-figure premises: with *AoB*, *AaC* this gives *AoB*, *CiA*, two particular premises. The second pair *AoB*, *BaC* is already in the first figure, but the universal premise *BaC* is 'about the minor extreme'; this also holds for the first pair. As Aristotle notes, he has proved (in A 4–6) that in a deduction in the first or second figure with *only* one universal premise, the universal must be the major premise.

60b28. 'this is the way . . .': i.e., it must be the major premise that is negative.

Chapter 11

61a18–21. 'A deduction through an impossibility': Aristotle here defines the subject which he investigates in Chapters 11–13. His definition implies that his subject is not the general technique of proof through impossibility, but a transformation which can be applied to existing deductions in the figures. There is a sort of formal similarity between this transformation and both circular proof and 'conversion': in each, we begin with a deduction p, $q \vdash r$ and produce another $f_1(r)$, $f_2(p) \vdash f_3(q)$, where f_1, f_2, f_3 are transformations applied to categorical premises (including conversion, 'contrary' conversion (or contradiction), 'opposite' conversion, and identity). Note that deduction through impossibility as here defined, unlike circular proof and 'conversion,' is a logically valid rule of inference. Aristotle's examination, however, appears, at least in part, intended to *prove* its validity rather than apply it. These Chapters should be compared with A 23, 41a21–b1, A 29, and A 44, 50a29–38.

61a21–27. By 'the way of taking premises is the same' (*hē autē lēpsis amphoterōn*) Aristotle means that the premise pair produced in a 'conversion' always corresponds to an identical pair produced in a deduction through impossibility, as the example (61a27–31) illustrates. The difference between 'conversion' and proof through impossibility may be seen as dialectical: conversion is a response to a deduction already constructed by someone else, whereas a deduction through impossibility is a way of generating an argument originally. Compare this account with A 23, 41a23–32; A 44, 50b32–38; B 14, 62b29–38. Note that for Aristotle, that statement the contradictory of which is deduced is not a premise of the *deduction*, though it is a premise of the *argument*.

61a27–31. 'For instance, if A': this example, which is supposed to show that there is a deduction through impossibility corresponding to every 'conversion,' is unclear on one point. Suppose that we have a deduction *AaC*, *CaB* ⊢ *AaB*. Aristotle has shown that this may be 'converted' either contrarily, giving *AaC*, *AeB* ⊢ *CeB*, or oppositely, giving *AaC*, *AoB* ⊢ *CoB*. Aristotle appears to be saying that there is a deduction through impossibility corresponding to *each* of these: one in which the assumption is *AeB* and one in which it is *AoB*. It is true, of course, that either of these leads to a contradiction with *CaB*. However, when we appeal to the 'impossibility' to conclude the 'opposite' (*antikeimenon*) of the assumption, we get *AiB* and *AaB* respectively. Aristotle may, at one time, have erroneously thought that a proof through impossibility of the first sort could actually establish, not *AiB*, but *AaB*: cf. *Posterior Analytics* I.26. (Note, incidentally, that by Aristotle's reckoning, these deductions through impossibility are in the *second* figure: the figure of a deduction through impossibility is the figure of its contained deduction.)

61a34–62b24. In the remainder of 11–13 Aristotle follows a set order of investigation. For each figure, he asks, in turn, how a deduction through impossibility may be constructed in that figure for a given categorical sentence type. He answers this by determining how the contradictory of the given categorical type may appear as a premise in the figure in question. However, Aristotle investigates possible deductions using the *contrary* of the intended conclusion as an assumption as well as the contradictory. In each case, he determines two things: (1) if the contained deduction is in the given figure, which premise (major or minor) should the assumption be? (2) should one assume the contrary or the contradictory of the conclusion? Aristotle's discussion is easier to follow if it is borne in mind that he consistently uses 'assume' (*hupotithenai*, *hupokeisthai*) or 'set down' (*keisthai*) of the premise used as *reductio* hypothesis and 'take' (*lambanein*) of the other premise. (In many cases, he uses no verb: I have filled in the blanks in accordance with his practice.)

The details of his investigation suggest that although he has *stated* the general logical principle on which proof through impossibility rests, he regards that principle as in need of a proof, which he gives by examining all possible cases.

61a35–b10. 'A universal positive': Aristotle considers two possible ways of deducing *AaB* through an impossibility: assuming its contradictory *AoB* or its contrary *AeB*. Neither of these can serve as the minor premise of a first-figure deduction; *AeB* can serve as the major premise, but deducing a contradiction only gives us the falsehood of *AeB*, not the truth of *AaB*. (His argument is curiously elaborate and indirect: why not say straightaway that assuming *AeB* will not work for this reason?)

61a38–39. 'from whichever side' (*hopoterōthenoun*): i.e., from the side of the predicate term (*CaA*) or from the side of the subject (*BaD*).

61a40. 'for in this way': to get a first-figure deduction with *AxB* as a premise, we must add either a major premise *CxA* or a minor premise *BxD*.

61b11–19. Aristotle's treatment of *i* conclusions in the first figure is representative. He shows: (1) we may use the contradictory assumption (*e*) as the major premise of *Celarent* or *Ferio*; (2) we cannot use the contradictory assumption as the minor premise; (3) we cannot use the contrary assumption (*o*) at all. He notes in summary that 'it is the opposite [sc. contradictory] which must be assumed [rather than the useless contrary].'

61b19–33. 'Next': the next case, that is, proving an *e* conclusion.

61b24–30. 'And if the contrary': if, in trying to prove *AeB*, we assume its contrary *AaB*, we may indeed come up with a first-figure deduction having a false conclusion, but the falsehood of the assumption *AaB* does not entail the truth of the desired *AeB*. Once again, Aristotle goes by a roundabout path to show this.

62a2–8. 'But when this has been proved': this difficult text is aimed at showing that we cannot prove an *o* conclusion with an *i* assumption. This would appear to be a mistake: Aristotle normally holds that an *e* sentence entails its subcontrary, so that proving *AeB* is sufficient for proving *AoB*. His objection here is that this will, in effect, go too far in a case in which *AoB* is strictly true, i.e., *A* belongs to some but not every *B*. Making the *i* assumption will 'reject what is true in addition,' that is, the implicit *i* sentence. With somewhat convoluted reasoning, Aristotle adds that the *i* assumption cannot lead to an impossibility because then it would be false, but we have supposed in this case that it is true. It is hard to see how to make this fully coherent: his point should be that it is not *necessary* to suppose the 'contrary' in this case, even if it is sufficient.

In 62a4–5, I read *ou para tēn hupothesin sumbainei to adunaton* ('[neither] does an impossibility follow as a result of the assumption'), with the majority of manuscripts, rather than Ross's conjecture *ouden para tēn hupothesin sumbainei adunaton* ('nothing impossible follows as a result of the assumption'). Aristotle often uses the phrase 'the impossibility' (*to adunaton*) without implying that some particular impossibility is being referred to (as in 'by means of an impossibility,' *dia tou adunatou*). Compare the analogous use of 'the necessity' (*to anankaion*) to mean 'the conclusion' even when there is no conclusion. The sense is 'an impossibility, *if there is one*, does not follow. . . .' Manuscript *n*, which has *oude* rather than *ou*, actually supports this better: 'the impossibility does not *even* follow as a result of the assumption.'

62a9–10. 'not to belong to some': on the expressions 'not to every' (*mē panti*) and 'not to some' (or 'to some not': *tini mē*), see e.g., 27a36–b3 and the associated Note. It is impossible to reproduce in English the important fact that the 'not' in one case comes *before* the term of quantity and in the other case *after* it.

62a11–19. 'It is evident': this passage gives two very different sorts of reasons for always assuming the contradictory rather than the contrary of the desired conclusion in a deduction through impossibility. First, Aristotle makes the logical point that only in this way does showing the assumption false always entail that the desired conclusion is true. His second point, however, is a matter of what is 'accepted' (*endoxon*): the term is an important one from the *Topics*, where it is defined to mean something like 'reputable' or 'received' (in common use, it means 'famous'). Aristotle's point seems to be that people, in general, will accept the inference from rejecting the contradictory of a statement to asserting that statement, though the same does not hold for the statement and its contrary.

Chapter 12

62a36–37. 'the same as in the case of the first figure': that is, no conclusion will be possible (cf. 61b17–19).

Chapter 13

62b25–28. Aristotle's summary of his discussion of deduction through impossibility indicates that at least one of his main concerns is to show that in such deductions it is the contradictory, not the contrary, which must be assumed. From our perspective, it is part of the *definition* of *per impossibile* deduction that the contradictory is what is assumed. This points to the fact that much of Aristotle's terminology derives from an existing dialectical practice. Argument through impossibility was a well established practice in philosophical and mathematical circles. However, to judge by the present discussion, some aspects of that practice were (as Aristotle discovered) indefensible on logical grounds. In concluding that one must assume the contradictory, not the contrary, of what one wants to prove through impossibility, he is recommending a refinement in a received procedure.

Chapter 14

62b29–38. This account may be compared with A 23, 41a21–b1, and A 44, 50a29–38. For the sense of 'familiar' (*gnōrimos*), see *Posterior Analytics* I.1–2 (compare also the Notes on B 16). This remark applies to the conclusion of the deduction itself (i.e., of the entire deduction in the probative case, or of the contained deduction in a deduction through impossibility). The point is that we do not even need to know in advance what the conclusion of a probative deduction is (since it is deduced from the premises), whereas we must know in advance that the 'conclusion' of the contained deduction is false. The remark that deduction through impossibility applies equally to negative and affirmative statements is not trivial: *Posterior Analytics* I.26 seems to associate it with negative statements only (cf. 63b19–21).

62b32. 'More precisely': this translates *men oun*, which as Ross says here 'introduces a correction.'

62b35–36. 'believe in advance' (*prohupolambanein*): the majority of translators take this word to mean 'assume in advance,' and LSJ lists only that and closely related meanings (significantly, all their citations are from Aristotle). But *hupolambanein* usually means 'believe' or 'conceive,' not 'assume' (cf. 64a9 below). Other occurrences of *prohupolambanein* in Aristotle (*Posterior Analytics* I.1, 71a12; *Rhetoric* II.21, 1395b6, 11; *Poetics* 25, 1461b1) concern understanding or believing something beforehand. In the *Poetics*, Aristotle is talking about how a poet's words are to be understood, and quotes a criticism of those who *start out* with an improbable interpretation; in the *Rhetoric*, the subject is really what people *already believe* (i.e., their prejudices); and in the *Posterior Analytics*, Aristotle is discussing what one must 'know in advance' (*proginōskein*) in scientific instruction. The present passage, in fact, closely parallels the *Posterior Analytics*. What Aristotle is talking about is not whether one need make an *assumption* beforehand, but whether one need *have any belief* about whether one's premises are true. Rolfes's translation is similar to mine ('man braucht nicht im voraus zu *wissen*, daß [der Schlußsatz] gilt oder nicht gilt'), although 'know' ('*wissen*') is probably too strong.

62b38–63b21. 'Everything concluded': The remainder of B 14 is a more elaborate proof of the claim made in A 29, 45a23–b11, that probative proof and proof through impossibility are interchangeable.

62b40. 'through the same terms': the Aldine edition adds 'but not in the same figures' here.

63a1. 'the true conclusion' (*to alēthes*): that is, 'the conclusion which is true' (on Aristotle's analysis, the 'conclusion' *deduced* in a proof through impossibility is the *false* conclusion of the contained deduction). This sentence could be paraphrased 'when the contained deduction in a proof through impossibility is in the first figure, then the corresponding deduction with true premises will be in the middle or the last,' and similarly for other cases.

63b12–13. 'it is also possible': the text here seems to me to be corrupt. Some manuscripts read 'it is also possible to prove each of the problems through the same terms probatively and through an impossibility.' Now, what Aristotle has just shown is that whenever a conclusion has been deduced through impossibility, then that same conclusion could also have been deduced probatively using the same terms: thus, 'and through an impossibility' seems not to give the right sense and Waitz and Ross accordingly reject it. But while the troublesome phrase cannot be right, the result of omitting it is, at any rate, a rather elliptical sentence. (The phrase might, in fact, be a corrupt form of the needed supplement: perhaps ***hōs kai dia tou adunatou***?)

63b16–18. 'the same deductions . . . by means of conversion': the 'conversion' meant is the procedure of B 8–10.

63b19–21. 'separated off': i.e., proof through impossibility is not limited to proving any particular type of categorical sentence.

Chapter 15

63b22–30. A deduction 'from opposite premises' is a deduction having as its premises some statement and its opposite (either contrary or contradictory). Premises of this sort will, of course, have the same subject and predicate respectively, and thus the premise pair will have only two distinct terms: the middle and a single 'extreme.' If a deduction is possible, it will be in either the second figure (with the common predicate as middle) or the third (with the common subject as middle). In his opening statement, Aristotle presents a surer understanding of opposites than found in B 8–10 (*i* and *o* statements are at once dismissed as merely 'verbal' opposites). The Chapter also seems to be largely independent of B 1–14.

Aristotle does not say what the purpose of these investigations is. They may be related to the dialectical game of the *Topics* in which the goal is to drive one's opponent into a contradiction (cf. 64a33–37 below), and there may also be some connection with the contents of B 2–4 (cf. 64b7–27). The most evident connection, however, is with the discussion of inconsistent beliefs in B 21.

63b31–39. Aristotle's argument here is curiously indirect: it would be simpler just to point out that opposite premises cannot occur in the first figure since the middle term must occur as subject of one premise and predicate of the other, from which it would follow that all three terms of the deduction would have to be the same.

64a11. 'converted in respect of the terms' (*epi tōn horōn*): cf. 64a40–b1, 64b3. The qualification may be intended to differentiate the sense of 'convert' from that in B 8–10.

64a16–17. The case in which 'the terms below the middle' (i.e., the extremes) are 'as a whole to a part' includes premises like 'Every science is good/ Medicine is not good,' which are not opposites. However, since medicine is 'part' of science, Aristotle replaces the second premise with the associated particular premise 'Some science is not good.'

64a33–37. 'We should take note': this passage suggests that the investigation of deductions from opposite premises has some sort of dialectical importance. It might be asked: whoever would try to argue from blatantly inconsistent premises like these? Aristotle answers: we can get the same result by deducing one of the premises from other things, or we might (as explained in *Topics* VIII.1) get our respondent to accept them if he is inattentive or we are skillful.

64a37–b6. 'And since': this passage is somewhat out of place. The point is that there are six possible opposite-premise combinations: *ae*, *ao*, *ie* and (by 'converting' the premises) *ea*, *oa*, *ei*. Ross assumes that Aristotle is talking specifically about the second figure, which leads to some difficulties in understanding the passage. But all Aristotle probably means is that these six are all the relevant combinations, and that he has investigated all six for both figures, even though he does not explicitly mention all of them in 63b31–64a32. He does appear to omit *ei* and *ie* cases, though he has discussed *Festino* implicitly at 64a12–13, *Ferison* at 64a27–30.

64b7–8. 'as was explained earlier': in B 2–4.

64b9–10. 'contrary to the subject' (*enantios . . . tōi pragmati*): *pragma* could mean 'thing' or 'fact' as well, and those senses might not be out of place here given that Aristotle regards the law of noncontradiction as a general truth about *things*. However, all he probably has in mind here is the much humbler point that the predicate deduced is contrary to (or at any rate inconsistent with) the *subject term* of the conclusion, as in his examples.

64b11–13. 'from a contradiction': Aristotle's point is that a pair of opposed premises constitutes a 'contradiction' (affirmation-denial pair), and the two parts of such a contradiction cannot simultaneously be true. The mention of subject terms takes account of the additional cases in which we have, not two exact contradictories or contraries, but premises in which the subject term of one is a part of the subject term of the other. (I thus take *kai* in *kai tous hupokeimenous horous* . . . as epexegetical.)

64b13–17. The 'trick arguments' (*paralogismoi*, 'paralogisms') Aristotle has in mind here are evidently arguments through impossibility. The sense in which deductions from opposite premises are 'contrary' has just been explained (64b11–13). The example 'not odd if it is odd' *may* be connected with a Greek proof of the incommensurability of the diagonal: see the discussion of the next Chapter. It may also be important to note that the most celebrated paradoxical arguments of Greek philosophy (Zeno's arguments about motion) took the form of arguments through impossibility.

64b17–27. The arguments to self-contradictory conclusions discussed here almost certainly have their home in the environment of dialectical refutation: as Ross observes, Plato's dialogues contain many examples. Aristotle distinguishes three cases: (1) getting the contradiction from a single deduction, (2) assuming one part of it and getting the other through a deduction, (3) getting both parts through deductions. The first case, which evidently involves a deduction having a premise with a complex and self-contradictory predicate such as 'white and not white,' may appear to be a merely formal possibility, but *Posterior Analytics* I.11, 77a10–21, seems to concern just such arguments (this is a difficult text to make sense of). The remaining two cases are those of 64a33–37. In 64b23 after 'belief' most sources add 'and not belief,' which would make Aristotle's example illustrate taking a contradictory 'straightaway.' However, as the example is developed it clearly illustrates case (2), in which one contradictory is 'taken' and the other obtained through deduction. It would be quite in harmony with Aristotle's practice to use 'take in addition' (*proslambanein*) of an additional premise like this, but not in connection with a self-contradictory premise such as 'every science is belief and not belief.' I accordingly follow Ross in rejecting these words.

64b24. 'the way that refutations are effected': this might be a reference to 62a40–b2.

Chapter 16

64b28–30. The traditional Latin translation for the subject of this Chapter (in Greek *to en archēi aiteisthai*) is *petitio principii*, 'asking for the starting point'; 'begging the question,' its traditional English 'translation,' bears only a remote similarity to Aristotle's phrase (and in my opinion, it is really nonsensical in modern English). Aristotle has in mind an argumentative or dialectical situation in which one participant is required to prove something proposed (the 'initial thing': *to en archēi*, *to ex archēs*). The proof is to be constructed by asking questions of the other participant. Aristotle clarifies the sense of the phrase by adding an explanatory 'or taking' (*kai lambanein*: *kai* is epexegetical): as he tells us in A 1, 24a22–25, the difference between dialectical and demonstrative premises is that the dialectician asks while the demonstrator takes (cf. Mignucci, 661–662). 'Asking for the initial thing' in its most straightforward form is, then, just putting the very thing to be proved to one's respondent as a question. To judge by Aristotle's remarks, here and elsewhere, the phrase was a term of art from early on in the history of institutionalized dialectic.

I have elected to translate *aiteisthai* with 'ask' rather than the more conventional 'postulate.' 'Ask' is what *aiteisthai* means in ordinary Greek; indeed, our use of the word 'postulate' simply descends from the Romans' use of Latin *postulare* ('ask') as a translation of this very verb in such contexts as Aristotelian dialectical terminology and subsequent philosophical and mathematical usage. Thus, to translate as 'postulate' is, in a way, not to translate but to encode.

One other point about the translation: the phrase 'to grasp its family, so to speak' translates *hōs en genei labein*. Most translators take the word *genos* (here in the dative *genei*) to be the technical term 'genus' and thus interpret the phrase as something like 'to grasp it in its genus' (whatever that may mean) or 'speaking generally'; indeed, Bonitz 150b32–33 gives this passage as the sole authority for such a meaning in Aristotle. But *en genei* is an ordinary Greek expression meaning 'related to' or 'in the same family as'; I have so taken it here.

64b30–34. 'several ways': compare this account of types of failure to demonstrate with the definition of 'demonstration' in *Posterior Analytics* I.2. Note that here Aristotle distinguishes between being 'prior' and being 'more familiar' or 'better recognized' (*gnōstoteron*). The latter term is often translated 'better known,' but it is clearly intended here as a synonym for *gnōrimōteron* ('more familiar') in the *Posterior Analytics*.

64b34–65a37. 'However, since some things': both the content and the purpose of this discussion of 'asking for the initial thing' are difficult to determine with certainty. We may divide the argument into three sections: 64b34–65a9, 65a10–25, and 65a26–37. In the first, Aristotle gives us a general definition of *to ex archēs aiteisthai*; in the second, he discusses the application of this to deductions in first-figure *Barbara*; and in the third, he expands the discussion to apply to deductions with negative premises or in other figures. It is the second of these sections which raises the most difficulties, both as to its overall meaning and with respect to textual details. We can, however, form a reasonable interpretation of it if we first understand the sections which surround it.

The definition Aristotle gives is 'trying to prove through itself that which is not familiar through itself' (*mē to di' hautou gnōston di' hautou tis epicheirēi deiknunai*). He later varies 'not familiar through itself' with 'not clear through itself' (65a25), and the immediately preceding sentence gives us 'not of such a nature (*pephukos*) as to be recognized (*gnōrizesthai*) through itself' as another equivalent. This is, as noted above, the language of the theory of demonstration in the *Posterior Analytics*, and it is specifically tied to Aristotle's own view that there *are* certain things which are 'familiar through themselves' and not in need of, or susceptible of, demonstration. It is somewhat surprising to see this offered as a general definition of 'asking for the initial thing,' given the meaning of that expression in dialectical practice. It is also difficult to imagine what 'trying to prove something through itself' would be. Would that simply consist of *asserting* it, or perhaps asserting it together with the claim that it needed no proof? But in that case it is hard to see how the question of 'asking for the initial thing' comes up as a question about an argument. We would have to suppose our demonstrator to say, when asked to prove X, 'There is no need to prove X; it is of such a nature as to be evident through itself.' It seems quite beside the point to respond to this with 'But you are asking for what it was required to prove.'

The critical point, I think, is that 'asking for the initial thing' is typically a matter of *surreptitiously* introducing the thing to be proved among the premises. This can be done in several ways, e.g., by substituting synonyms for the terms in the conclusion and hoping that our opponent will not notice. As a dialectical criticism, the point of 'you are asking for the initial thing' is something like this: 'you are supposed to be deducing the required conclusion from *other* premises, not just asking me to concede it.' Consequently, it embraces not only the blatant case in which the one putting the questions just turns the intended conclusion into a question and asks for it, but also, cases in which the questioner asks something which, to put it somewhat loosely, no one would concede who would not already concede the conclusion.

And this is precisely where the problem for analysis arises: how are we to describe such cases? There is no sharp line between blatantly asking for the desired conclusion, asking for it in a disguised form, and asking for something which, on reflection, we might regard as equivalent to it, or equally hard to swallow. What Aristotle offers us here is a general characterization of what *can* legitimately be asked for, employing his own notion of the 'priority' of one premise to another: there are some things which, on his view, are *by nature* prior to others, and it is an error to ask someone to concede what is posterior in trying to prove what is prior. Thus, in 64b38–65a1, Aristotle distinguishes the blatant case of asking 'directly' (*euthus*) for the conclusion, but the fact that he says nothing more about it suggests that he considers this merely a possibility and not the interesting case.

But there is another aspect to this discussion. Many details make it clear that Aristotle has circular proofs (in the sense defined in B 5) in mind. In modern use, the expressions 'arguing in a circle' and 'begging the question' are roughly interchangeable, which may contribute to our own inability to see that these are, for Aristotle, completely different things. A circular deduction for him is an extended structure of deductions in which each premise also appears as a conclusion; 'asking for the initial thing' is, instead, a dialectical matter. However, Aristotle himself associates the two closely by arguing in *Posterior Analytics* I.3 that those who try to prove everything using circular demonstrations are really just 'asking for the initial thing.' He tells us, there, that circular proof is just proving that 'when A is, then A is' (73a4–6, 72b32–35); he uses almost exactly the same words about cases of 'asking for the initial thing' here in 65a7–9. The connection is confirmed in 65a10–25, where the cases of 'asking for the initial thing' studied turn out to be identical in form to the circular deductions of B 5.

Now, what Aristotle seems to be doing here is the converse: treating at least a large number of cases of 'asking for the initial thing' as circular deductions. We can, I think, make sense of this if we remember his initial characterization of 'asking for the initial thing' as 'trying to prove through itself what is not naturally proved through itself.' The only plausible case of such an attempt which emerges from Aristotle's account is just exactly the circular demonstrator's attempt at proving things from one another. Thus, two cases of 'asking for the initial thing' emerge: the blatant or 'direct' case, and the case of the circular demonstrator. It is, in fact, not unreasonable to suppose that these are the only cases possible. Suppose p is prior to q and that it is natural to prove q from p, but nevertheless, possible to use q in deducing p. This can only happen if p and q can be used in deducing one another, i.e., in a circular demonstration.

65a4–7. Commentators generally take Aristotle's reference to 'those who think they draw proofs that there are parallels' to concern attempts to prove the parallel postulate: for discussions, see Heath *1926*, I.191; Ross 462–463. Although the verb *graphein* literally means 'draw,' Aristotle frequently uses it to mean 'prove' in geometrical contexts (see the Notes on 46a8 and 41b14). I try to capture the sense that a diagram is probably always presupposed with the (perhaps intemperate) translation 'draw proofs.'

65a10–25. There are difficult questions about the language in this passage, but the overall argument seems clear enough. Aristotle considers two cases. In the first, the putative demonstration is the deduction

(1) $AaB, BaC \vdash AaC$.

If one of the premises, for instance AaB, is 'equally unclear' as the conclusion AaC, then this fails to be a demonstration. But Aristotle now adds a second possibility: suppose that B and C 'convert,' so that we also have as a premise

(2) CaB.

We then have a deduction in *Barbara* together with the converse of one of the premises. Accordingly, we can also deduce the other premise:

(3) $AaC, CaB \vdash AaB$.

Note that this is exactly the pattern of a 'circular proof' as dicussed in B 5. Aristotle tells us that in this case, the would-be demonstrator is 'asking for the initial thing.' Similarly, if BaC should be 'equally unclear' as the conclusion, then (1) would fail to be a demonstration. However, if A and B should convert, so that we also have as premise

(4) BaA,

then it is also a case of 'asking for the initial thing.' Aristotle tells us that this is 'for the same reason' (65a3). By analogy with the first case, this must mean that (4) and the conclusion *AaC* permit the deduction

(5) *BaA*, *AaC* ⊢ *BaC*,

again in circular-proof fashion.

But while the argument of this passage seems clear enough, Aristotle's language raises problems in a number of points. The following Notes address these one by one.

65a14–15. 'one belongs to the other' (*thateron thaterōi huparchei*): this phrase, and its companion '*A* follows *B*' in 65a22–23, are difficult to understand. Aristotle defines three cases in which 'asking for the initial thing' arises: (1) *B* is the same as *C*, (2) *B* converts with *C*, (3) 'one belongs to the other.' In 65a21–23, where the argument is exactly parallel, he instead presents the cases corresponding to (2) and (3) as subcases of the case corresponding to (1): '*A* [is] the same as *B* because *A* either converts with or follows *B*.' In each instance, Aristotle's third case (or second subcase) seems absurdly wide: '*B* belongs to *C*' is simply the premise itself, in the first example, as is '*A* follows *B*' in the second, and one term's following another is hardly a plausible reason for calling them identical.

Ross undertakes to solve the problem in 65a15 by reading *enhuparchei*, 'belong in,' i.e., 'be essentially predicated of,' which just may have been what Philoponus read (so Mignucci: 'l'uno è presente nella definizione dell'altro'). He then must treat 'follows' in 65a22 as equivalent to this and regard both as indicating a sort of 'partial identity.' But this is really of no help. Aristotle would have to be saying that if *B* is essentially predicated of *C*, then using *BaC* as a premise to deduce *AaC* is 'asking for the initial thing,' and by these standards many (perhaps all) of his paradigm demonstrations would be ruled out.

The evidence indicates, instead, that Aristotle is in each case thinking only of identical or convertible terms. First, as noted above, the argument closely parallels the discussion of circular demonstration, which is only possible for convertible terms. Second, when Aristotle offers a reprise of his results a few lines later in 65a28–29, he only mentions identity. Even if we do take essential predication to indicate a kind of partial identity, it is convertibility that is critical to Aristotle's argument; and partially identical terms are not convertible. We must, therefore, suppose that each of these problematic phrases somehow expresses identity. The phrase 'one belongs to the other' might, with strain, be taken to mean '*each* belongs to the other'; 'follow' would then need to be elliptical for 'follow each other,' which is not very plausible. It is, I think, impossible to accept Hintikka's argument (*1973*, 53–55) that *hepesthai* sometimes expresses equivalence, congenial as this would be.

65a17–19. 'if he converted it' (*ei antistrephoi*): like Tricot and Rolfes, I take 'convert' to have a transitive sense here. In the phrase 'as it is, this prevents him, but not the type of argument,' many commentators take 'this' to be 'the fact that *BaC* does not convert.' But there is no such fact: the very case in view is the case in which *B* and *C* are identical or clearly convertible. Instead, what Aristotle means is that it is only the arguer's failure to convert (that is, take *CaB* as a premise) that prevents him from deducing *AaB*, not anything about the actual relationship of the terms. The meaning of 'the type of argument' (*ho tropos*) is probably something vague like 'the way the argument works.'

'if he did this, he could do what was stated': that is, if he converted *BaC* to get *CaB* and deduced *AaB* from this and *AaC*, then he would be able to carry out 'what was stated' in B 5 in the account of circular deductions. The phrase continues with an explanation of what 'what was stated' is (*kai* is epexegetical). 'Convert through three terms' means 'as the result of a deduction (which requires three terms)': cf. 57b17. The majority of interpreters suppose, instead, that 'what was stated' refers to the definition of 'asking for the initial thing' in 64b36–38. But while Aristotle certainly agrees with that, I believe the reference to B 5 is his immediate point.

65a26–35. This section extends the account of asking for the initial thing to other deductions and other figures. Aristotle is obviously relying on the results of B 5–7 (for a very full discussion of the details, see Mignucci, 666–673).

65a29. The phrases 'the same things . . . to the same thing,' 'the same thing . . . to the same things,' which summarize the discussion of 65a10–25, are clear in meaning and establish that it is identity and convertibility that are in question in that section. Note that Aristotle here seems to equate coextensionality or convertibility with identity.

65a30. 'in both ways': i.e., either with the convertible terms both as predicates or with them both as subjects.

65a35–37. 'Asking for the initial thing': This remark appears to have been tacked on to the discussion (but cf. A 30, 46a8–10).

Chapter 17

65a38. This Chapter concerns a type of objection which may be voiced to a proof through impossibility: 'the falsehood does not follow because of this' (*ou para touto sumbainei to pseudos*). It is clear, again, that the phrase is not Aristotle's coinage but part of the currency of his day: his purpose here is to accommodate it in his deductive theory, and also to recommend a more precise sense for it, much as he does with 'asking for the initial thing' in the previous Chapter. Aristotle first takes note that this objection may properly be used only in criticizing proofs through impossibility, not in attacking direct deductions which happen to have negative conclusions or in rejecting a statement by proving its contradictory.

65b1–4. 'For unless the argument had come to a contradiction': this is a rather expansive translation of *mē antiphēsas*, 'unless having contradicted.'
65b3–4. 'it does not suppose what it contradicts': that is, a probative argument does not include a supposition which is then contradicted by the conclusion of a 'leading away to an impossibility.' The text is uncertain at this point: I have followed Ross's *ou gar tithēsi ho antiphēsin* and taken the subject of both verbs to be 'the argument.' Other well attested readings, however, include 'it does not suppose the contradictory' (*tithēsi tēn antiphasin* and 'the person who is going to contradict does not suppose it' (*ou gar tithēsi ho antiphēsōn*).
65b8. 'assumption': I use this translation here, and at 65b14, 66a2, 66a8, for *thesis*. When Aristotle makes frequent use of the word *hupothesis* in a passage, he occasionally omits the prefix *hupo*, and *hupothesis* occurs in this Chapter with considerable frequency: 65b11, 14, 22, 28, 32, 34, 66a3, 12. Compare the similar use of *keisthō* for *hupokeisthō* at B 12, 62a23.
65b9–12. This final account of the 'not because of this' phrase shows the important issue. In a deduction through impossibility, an assumption is rejected because *it* leads to an impossibility if assumed. But, in general, there will be several premises to a deduction; and a deduction through impossibility does not, strictly speaking, tell us which of its premises to reject, but only that we cannot maintain them all. We might then say that the deduction through impossibility permits us to deduce the denial of any of its premises by retaining the remainder. The objection 'not because of this' introduces a restriction on this move: we cannot use a deduction through impossibility to prove the denial of one of its premises unless no impossibility follows from the *remaining* premises. Thus, Aristotle here adopts a position broadly similar to modern relevance logic.
65b13–21. Aristotle's first case is simply the importation into the argument of an unrelated deduction of an impossibility (presumably by importing its premises). For the sense of 'unconnected' here, cf. A 25, 42a21. Heath *1949*, 30–33, sees in the example a reference to an alternative proof of incommensurability. Such a proof may well have existed; but if that is what Aristotle has in mind, then evidently he regarded it as fallacious. It seems to me much more likely that the example he has in mind is purely fanciful. Presumably, he envisions someone who, first, assumes that the diagonal of a square is commensurable with its side; then, imports one of Zeno's arguments against motion, bringing it to its impossible conclusion; and, finally, concludes that the assumption of commensurability is false since an absurdity has been deduced. The reference to the *Topics* is probably to *Sophistical Refutations* 5, 167b21–36, as Ross suggests.
65b21–32. Aristotle's second case concerns assumptions which are, in fact, 'connected' (that is, by a middle term) to the impossibility deduced, but nevertheless not the 'cause' of it. He allows that an assumption connected in this way with the impossible consequence may still fail to be the 'cause' of it. Therefore, the initial criterion, which amounted simply to being linked to one

of the terms of the premise to be rejected through a chain of terms, is too broad and needs refinement. We get a refined criterion in 65b32–40: each of the terms of the assumption must be connected with the impossible conclusion in 'the appropriate way.' Again, this is reminiscent of the efforts of modern relevance logicians to find a formal criterion (such as variable-sharing) for relevance.

66a1–15. This passage indicates the unsettled condition of Aristotle's thought on his subject. Conceding that even his revised criterion may fail, he tries for a final improvement, which amounts to saying that 'not because of this' means 'the assumption is merely a superfluous premise in the deduction.' The fit between this and the technical definition appealing to connections through middle terms is not spelled out, and it is not clear how it should or could be. It is also not clear that Aristotle's position is fully consistent with B 4, 57a40–b17.

66a13–15. For discussions of the mathematical example, see Heath *1949*, 29–30, and Mignucci, 679–681.

Chapter 18

66a16–24. A 'false argument' (*pseudēs logos*) is a deduction with a false conclusion; Aristotle's point is that in every such argument there must be a 'first' or 'highest' false premise from which the false conclusion results. Although related to the preceding discussion, this note does not depend on it and seems, in fact, to reflect a more primitive understanding. Especially significant is the assumption that in every argument with a false conclusion there is a single 'first falsehood': this seems completely oblivious of the difficulties Aristotle has just gone through in trying to define the 'relevant' falsehood in a deduction through impossibility. We may also note that letters are used in this section to denote premises, not terms as in B 17. In view of these details, I suspect that B 18 is an earlier study of the same question.

66a17. 'from two premises': the Greek text is actually 'from *the* two premises,' which might mean 'from the two premises as explained in the account of the figures.'

Chapter 19

66a25–32. This section and the one following are unusual in the *Analytics* in that they concern argumentative (or disputational) technique rather than proof. B 19 is reminiscent of *Topics* VIII, but it clearly presupposes the contents of *Prior Analytics* A. The term *katasullogizesthai*, 'be argued down' or 'be defeated in argument,' occurs nowhere else in Aristotle or other classical authors.

'Allowing the same thing twice' does not mean agreeing to the same premise twice, but rather conceding two premises with a term in common.

66a32. 'what argument we are defending' (*pōs hupechomen ton logon*): literally, 'how we are defending the argument.' The term 'defend' (*hupechein*) is a technical term of dialectic, indicating the opposite argumentative role to 'attack' (*epicheirein*): see *Topics* VIII.3, 158a31 (and cf. *Posterior Analytics* I.12, 77a40–b15). Note that *epicheirein* occurs in the next sentence (66a34).
66a37. 'without middles' (*amesa*): that is, premises should not be presented for acceptance in an order which makes it apparent that their terms form a chain.

Chapter 20

66b4–17. This Chapter gives a further application of Aristotle's deductive theory to argumentative practice (the assimilation of refutations to deductions indicates his aim of generalizing as far as possible). Here, something which 'gets an affirmative response' amounts to an affirmative premise, something which 'gets a negative response' to a negative premise. This usage makes sense in a dialectical context, since premises are always put as questions admitting a yes or no answer. In defining a refutation (*elenchos*) as 'a deduction of a contradiction,' the term 'contradiction' (*antiphasis*) probably has the sense 'contradictory of some assertion' (i.e., which one's opponent has maintained).
66b4. 'when . . . i.e.': the *kai* in *pote kai pōs echontōn tōn horōn* must be epexegetical if it is to make any sense.
66b9–10. 'what is proposed': i.e., 'proposed for refutation.' The Greek, *to keimenon*, is equivalent to *to prokeimenon* (Aristotle occasionally drops a prefix from a compound verb like *prokeisthai*).
66b16–17. 'the determination of a refutation and of a deduction are the same': 'determination' (*dihorismos*) might be taken to mean something like 'distinction' or even 'account.' It frequently means 'definition,' but that sense will not work here: Aristotle does not mean that 'refutation' and 'deduction' are synonymous, but that the results proved earlier about deductions also apply to refutations (presumably because a refutation is a *species* of deduction).

Chapter 21

In this Chapter, Aristotle wants to explain how it is apparently possible for someone to have both knowledge and ignorance about the same thing at the same time, in violation of the law of noncontradiction. His answer rests on a distinction of three kinds of cases of knowing. This Chapter should be compared with the discussion of 'ignorance with respect to a disposition' in *Posterior Analytics* I.16–17. The subject here is an important one for Aristotle. In *Metaphysics* IV he tries to argue that no one can have beliefs which contravene the law of noncontradiction; here, he tries to explain what appear to be examples of just such contrary beliefs. Perhaps more important still is the connection with the problem of 'weakness of will,' that is, the paradoxical fact that we sometimes seem to act consciously in disregard of our considered best judgments. Aristotle's discussion of this problem in *Nicomachean Ethics* VII (= *Eudemian Ethics* VI) recalls the present discussion in a number of its details and should be compared with it (see in particular VII.3).

66b18–34. Aristotle's opening reference to 'falling in error in connection with the position of terms' (*en tēi thesei tōn horōn apatōmetha*) strongly recalls the subject of A 33, which is 'being deceived . . . by the resemblance of the position of the terms' (*apatasthai . . . para tēn homoiotēta tēs tōn horōn theseōs*: 47b15–17). What Aristotle discussed there were arguments which appear to be deductions because the terms in them appear to be in relationships of predication although they are not. Here, he evidently resumes his earlier discussion of types of deception or error in reasoning. If indeed B 21 is a continuation of A 33, its otherwise intrusive appearance in Book B would be explained.

The principal difficulty with which Aristotle is concerned is this: Suppose that there are two sets of premises from which a certain conclusion can be proved, and suppose that someone believes the premises in one set but disbelieves those in the other. If we suppose that knowledge of what is demonstrable is just knowledge of premises from which it can be demonstrated, it then seems to follow that this person simultaneously knows and does not know the same thing.

66b20–23. 'the same thing . . . several things primarily': *A* belongs to *B* 'primarily' or 'first' (*prōtos*) if *AaB* and there is no term *C* such that *AaC* and *CaB* (or in other words, *AaB* is 'without a middle term,' *amesos*). For more on this see *Posterior Analytics* I.15. Aristotle treats 'belonging of itself' (*kath' hauto*) as equivalent to this. In *Posterior Analytics* I.16–17, he distinguishes cases of inferred ignorance involving unmiddled and nonunmiddled premises respectively. Here, he may only wish to rule out the possibility that any of the premises in the example is itself known or believed on the basis of a deduction.

66b22–30. Aristotle envisions two cases: In each, we suppose that *AaB*, *AaC*, *BaD*, and *CaD* are all true, so that we can deduce *AaD* with either *B* or *C* as middle term. In each case, suppose also that someone correctly believes *AaB*, *BaD*, and *CaD*, but mistakenly believes *AeC* (and therefore is in a position to deduce both *AaD* and *AeD* from premises he believes). The difference between the two cases is that in the first (66b22–26), the terms *B* and *C* are not 'from the same series' (that is, neither *BaC* nor *CaB*), whereas in the second case (66b26–29) they are (and in particular *BaC*). The term 'series' (*sustoichia*) means 'sequence of terms each of which is universally true of its successor.'

66b30–31. 'Based on these premises': the puzzle Aristotle raises here applies to both the previous cases. Aristotle's answer appears in 67a8–26.

66b34–67a5. Aristotle first responds to his problem by arguing that certain types of inconsistent beliefs are indeed impossible. In the case of terms not from the same series, 'the first premise is taken as a contrary,' so that the person in question would simultaneously have and not have the same belief, which is impossible.

Evidently, Aristotle regards the premises *AaB* and *AeC* as somehow contrary to one another, although it is not clear how. One suggestion is the following: He rephrases *AeC* as '*A* belongs to none of what *C* belongs to' and *AaB* as '*A* belongs to what *B* belongs to.' Now, given *CaD* and *BaD*, we can take *D* as an instance both of 'what *C* belongs to' and 'what *B* belongs to.' This would bring the discussion of this case more into line with the subsequent discussion of the *Meno* argument and universal *versus* particular knowledge (it is not clear why Aristotle could not offer the same solution for this case as he does for the later one).

The argument in the *Meno* rests on a puzzle in Plato's *Meno* (80d–e) which purports to show that we cannot seek either what we know (for we already have it) or what we do not know (for we could not tell if we found it). Aristotle refers to this problem more than once in developing his theories of knowledge: see *Posterior Analytics* I.1, 71a29–30, and the discussion in Ferejohn *1988*.

67a5–21. Knowing the universal while failing to recognize one of its instances (e.g., because we do not even know that *this* particular exists) does not entail self-contrary beliefs, since there are two ways in which we can have knowledge of all of something. This distinction closely corresponds to that made in *Posterior Analytics* I.1.

Several times in this passage, Aristotle uses an idiom that is ambivalent between 'know' in the sense of *connaître* and know in the sense of *savoir*: 'know *x*, that . . .' (e.g., *oide to G, hoti duo orthai*, 'he knows C, that it is two right angles,' or similarly *agnoein to G hoti estin*, 'he does not know C, that it exists'). I have tried to preserve this ambiguity with a somewhat barbarous English construction. (The same locution is found a number of times in Plato's *Meno*, e.g., 71a5.)

67a12–13. 'ignorant that C exists': more literally, 'not-knowing C, that it is' (*agnoein to G, hoti estin*).

67a20. 'contrary states of knowledge': in Greek, simply 'contrary [knowledges]' (*tas enantias*, with the governing noun *epistēmas* clearly implied). I have tried to supply a more idiomatic English rendering. We might almost borrow a modern philosophical idiom and say 'contrary epistemic states.'

67a21–26. Applying this distinction to solve the problem in the *Meno*, we see that the advance knowledge required is only universal knowledge, never knowledge of particulars. This section very closely resembles *Posterior Analytics* I.1, 71a17–b8: see McKirahan *1983* for a discussion.

67a27–30. Aristotle here distinguishes two different types of knowledge of particulars: 'contemplating' them (*theōrein*) and knowing them 'in virtue of their peculiar knowledge' (*tēi oikeiāi* <*epistēmēi*>). 'Contemplation' or 'reflection' is in Aristotle associated with demonstrative science. It should be remembered here that, for him, the objects of science are unchangeable, unlike sensible particulars.

67a29–30. 'be in error about the particular' (*apatasthai de tēn kata meros*): this is Ross's conjecture (the manuscripts have *tēi*). He suggests that 'the particular' is 'the particular error' (*tēn kata meros apatēn*), noting that the verb *apatasthai* often takes its cognate noun as direct object, as at *Posterior Analytics* I.5, 74a6. However, it could also mean 'the particular knowledge' (in contrast to 'the universal' a few words earlier), in which case it would be an 'accusative of respect' ('in respect to the particular knowledge').

67a30–38. Aristotle now returns to the initial case of two inconsistent beliefs resting on deduction, offering his solution to the puzzle at 66b30–34). In this case, it is a failure to 'reflect simultaneously' (*suntheōrein*) that accounts for the apparent possession of contrary beliefs: the person in question knows that *AaB* and that *BaC*, but simply does not consider them together, and as a result, the inference to *AaC* is not made. It is, therefore, possible for this person to believe the contrary of *AaC*. The case given here seems to be exactly parallel to that in 66b34–67a5: the distinction, evidently, is that here the person's error is not a matter of holding two contrary beliefs, but rather two beliefs of different kinds (a universal belief and a particular belief).

67a38–b11. In addition to the distinction between universal and particular knowledge, Aristotle now distinguishes between possessing and exercising (*energein*) knowledge. These two distinctions are independent of one another: universal and particular knowledge can each be either exercised or possessed but not exercised (elsewhere, Aristotle refers to the latter as 'potential' knowledge). In the present case, he recognizes two instances of exercising knowledge, viz., the actual perception of sensible objects and the actual making of inferences (which apparently is automatic once the premises are 'considered together'). The failure to make the inference from 'Every mule is infertile' and 'This is a mule' to 'This is infertile' is a failure of the latter kind.

The distinction between knowledge as possessed and knowledge as exercised is a favorite theme for Aristotle (see e.g. *On the Soul* II.1, 412a10–11; *Metaphysics* V.7, 1017a35–b6, XIII.10, 1087a15–16; *Topics* V.2, 130a19–22; *Nicomachean Ethics* VII.3, 1146b31–33; *Eudemian Ethics* II.9, 1225b11–14).

67b12–26. This section does not further the discussion in the rest of B 21, although it does concern a relationship between beliefs and facts and the possibility of someone having inconsistent beliefs. It appears to reflect a less thorough familiarity with the issues than the preceding discussion; the call for further discussion at the end (67b26) may indicate that it is an earlier essay. The phrase 'the essence of good' (*to agathōi einai*: literally, 'the to be for good') is an Aristotelian usage with the approximate sense 'what it is to be for good' (for Aristotle, a definition of something is an 'account of its essence').

Aristotle's real concern here is the relationship between beliefs about identity and beliefs about predication. To begin with, he tells us that believing that being good is being bad is believing that being good is *identical to* being bad (and thus not a matter of believing that one is predicated of the other). He then argues that one cannot believe that *A* is identical to *B* without also believing that B is identical to A, appealing to an analogue of a deduction with convertible terms. Finally, he asserts that, in effect, inferential relationships are the same in belief and thought as in fact: if a set of premises entails a conclusion, then believing the premises entails believing the conclusion.

I have taken the example of believing that the essence of good is the essence of bad as merely an arbitrary example, with no further significance (Aristotle often uses concrete terms in discussions: cf. B 4, 57a36–b17). It seems probable, however, that Aristotle has in mind some type of situation in which it is argued that a person has inconsistent beliefs as a result of thinking that the same thing is simultaneously good and bad (which could perhaps form the basis for an argument in Platonic fashion that this person confuses the nature of goodness with the nature of badness). This, in turn, may lend further support to the suggestion of a relationship between this passage and Greek discussions of the problem of weakness of will.

Chapter 22

The miscellany of results in this Chapter may be intended to function as lemmas for the last five Chapters of the Book (B 23–27); otherwise, their provenance and purpose are somewhat obscure. Many of the results established here are established elsewhere in more complete form, and there are several puzzling or erroneous passages. B 22 thus appears to be an early fragment, written before the full study of deductions in Book A had been completed.

67b27–68a3. Here, as in B 5–7, Aristotle explores the result of supposing that one of the premises of a deduction is 'convertible.' However, the term 'convert' now has a different sense: a premise is said to 'convert' here if its converse (the result of exchanging subject and predicate terms) is also true (evidently, Aristotle has only true premises in view). If an *a* premise converts in this sense, then its terms convert in the sense of B 5–7 (that is, are coextensive). However, all (true) *e* premises convert in this way. As a result, the present notion of conversion appears to be not fully thought out, suggesting that the passage may be an earlier attempt at the same study given in B 5–7.

Aristotle first (67b27–32) shows that either premise of a deduction in *Barbara* (e.g. *AaB*, *BaC* ⊢ *AaC*) can be deduced from the converse of the conclusion together with the other premise:

(1) With the minor premise: *BaC*, *CaA* ⊢ *BaA*
(2) With the major premise: *CaA*, *AaB* ⊢ *CaB*

This result is, of course, identical with that established in B 5. Next (67b32–68a3), he turns to the case of 'not belonging' (i.e., a deduction *AeB*, *BaC* ⊢ AeC in *Celarent*), announcing that this is 'likewise.' Now, what Aristotle said initially was that when the extremes convert then the middle must convert with either of them. We would accordingly expect him to mean here that if the extremes of *Celarent* convert, then each of the premises must also: that is, that the converse of either premise can be deduced from the converse of the conclusion and the remaining premise. But Aristotle proved in B 5 that this is false: only the major premise can be so derived.

Aristotle's actual account is problematic. He first shows that the converse of the *conclusion* can be derived by converting the *major premise*:

(3) *BaC*, *BeA* ⊢ *CeA*

The deduction in (3) is second-figure (*Camestres*). The next case is difficult to follow and textually corrupt. Its first sentence appears in several forms: 'If B converts with C, then A also converts <with it>' (manuscripts A^1, B^1, G); 'if C converts with B, then it also converts with A' (A^2, B^2, n^2); 'and also if C converts with B' (n^1). The second of these, which is the most common reading, implies that we can get *CeA* (the converse of *AeC*, the original conclusion) by using the converse of *BaC*. But neither the pair *CaB*, *AeB* nor the pair *CaB*, *BeA* gives this conclusion. The first reading is ambiguous, since it does not say what A converts with, but since this construction usually means 'with the last term that occurred in the dative case' it would probably be C. But this is then the same case as the second reading. Ross, instead, combines both readings for 'if C converts with B, then A will also convert <with it>,' thus giving the deduction *CaB*, *AeC* ⊢ *CeA* (fourth-figure *Camenes*). Ross also finds a problem in the third case. The deduction is clearly *CaB*, *CeA* ⊢ *BeA*; but although Aristotle explicitly uses the converse of the minor premise (*CaB*), he only mentions the use of the converse of the conclusion (*CeA*). Ross accordingly inserts *kai* in 67b38 to get 'and if *in addition* (sc. to C converting with B, from the previous case) C converts with A.' He thus arrives at the following three deductions:

(3) *BaC*, *BeA* ⊢ *CeA* (*Camestres*)
(4) *CaB*, *AeC* ⊢ *CeA* (*Camenes*)
(5) *CaB*, *CeA* ⊢ *BeA* (*Camestres*).

Ross then must explain Aristotle's remark (68a1–3) that only the last of these 'begins from the conclusion,' i.e., has the conclusion (or its converse) as a premise: (4), after all, has the conclusion itself as a premise. According to Ross, Aristotle means that only the last uses the *converse* of the conclusion as a premise, as the two affirmative cases do.

This interpretation is strained in several respects. To begin with, (4) is a fourth-figure deduction, and even though Aristotle may recognize all the deductions in the fourth figure, he virtually never appeals to them. In addition, Ross's interpretation of 68a1–3 is unnatural: wanting to say 'does not begin with the *converse* of the conclusion (though perhaps with the conclusion itself),' Aristotle says 'does not begin with the conclusion.' It is also difficult to see either (3) or (4) as illustrating the claim that 'if the extremes convert, then the middle must convert with respect to each.' Since it is hard to imagine Aristotle writing this passage after having written B 5–7, I suggest that he wrote it before he had worked out all the details of his deductive system and that his second case simply contains a mistake: the received text, which gives the invalid *CaB*, *AeB* ⊢ *CeA*, is what Aristotle intends.

There are two further arguments for this: First, Aristotle seems to accept the similar invalid inference *CaB*, *AeB* ⊢ *AeC* in the *Posterior Analytics*, indicating that traces of an imperfect understanding of his own deductive theory persist in the treatises (see the discussion in Smith *1982*). Second, and more speculatively, we may make a good guess as to Aristotle's strategy here. Attempting to apply the conversion procedure of 67a27–32 directly to *Celarent* gives the premise pairs *CeA*, *BaC* and *CeA*, *AeB*; neither of these yields the converse of the other premise. Accordingly, he asks: what else might work? He notes that by converting one premise, he can get the converse of the conclusion:

(3) *BaC*, *BeA* ⊢ *CeA*

Next, he tries to do the same with the other premise, erroneously thinking that this works also:

(4) *CaB*, *AeB* ⊢ *CeA*

Finally, he notes that the first premise of (4) together with its conclusion yield the converse of the other premise:

(5) *CaB*, *CeA* ⊢ *BeA*

68a3–16. Bearing in mind that two terms are such that one or the other, but not both, belongs to everything if and only if they are contradictories of each other, we may restate the two results proved here as: if *A* and *C* convert with *B* and *D* respectively, and *A* is the contradictory of *C*, then *B* is the contradictory of *D* (68a3–8); if *A* and *C* are the contradictories of *B* and *D* respectively, and *A* converts with *C*, then *B* converts with *D* (68a11–16). Pacius observed that the example, which in the manuscripts appears between these two results as 68a8–11, actually illustrates the *second* point. I follow Ross's text in transposing these lines.

68a16–25. Aristotle appeals to the result proved here shortly afterwards (68b24–27). The remark in 68a20–21 that 'B will be said of all of those things of which A is said *except for A itself* (*plēn autou tou A*) is puzzling. Mignucci notes Kirchmann's suggestion that Aristotle may not regard convertibility as a criterion for identity of terms (699); but even if that is so (as it probably is), it is irrelevant here, since the question is precisely one of extension (and cf. the Notes on 65a14–15, 65a29). Since Aristotle is generally reluctant to treat terms as predicated of themselves, it may be that he means '*B* is predicated *just exactly* what *A* is predicated of, with the exception of *A* itself, of which *B* is also predicated'; but against the supposition that he wants to avoid 'self-predications' is his remark in the preceding sentence that *B* is predicated of itself. Ross suggests instead that *B* is not predicated of *A* (though coextensive with it), because *A* is the genus of *B* (but Aristotle does not indicate such a restriction).

68a25–39. The terms 'preferable' (*hairetōteron*) and 'more to be avoided' (*pheuktoteron*) played a major role in dialectic as Aristotle knew it; *Topics* III.1–3 is a collection of principles for determining what is preferable to, or more to be avoided than, what. This is related to the larger study of argument in two quite different ways. First, as *Topics* III.4–6 shows, Aristotle noted that results concerning these terms could be generalized to apply to any relative terms. Second, choice-worthiness and its opposite may be important to Aristotle because of his understanding of 'practical reasoning,' or reasoning concerned with means and ends and leading to action. According to *Nicomachean Ethics* VI.2, pursuit and avoidance play the same roles in practical reasoning as affirmation and denial in theoretical reasoning, suggesting a greater importance than at first appears for studies like the present one.

Aristotle's language in this section is frequently highly abbreviated; I have filled it out in what I think are fairly obvious ways, but the reader should beware.

68a26. 'preferable in the same way': that is, preferable *to the same degree.*

68a39–b7. This example, which simply illustrates the point made in 68a25–38, recalls the many erotic examples of Plato's dialogues.

Chapter 23

B 23–27 form a continuous investigation of several argumentative terms: 'induction' or 'leading up to' (*epagōgē*), 'example' (*paradeigma*), 'leading away' (*apagōgē*), 'objection' (*enstasis*), 'likelihood' (*eikos*), 'sign' (*sēmeion*). Aristotle's purpose is to bring 'absolutely any form of conviction whatever arising from whatever discipline (*haplōs hētisoun pistis kai hē kath' hopoianoun methodon*) under the umbrella of the deductive theory of A 1–22. As such, these Chapters form a natural continuation of A 1–44. The terms Aristotle discusses were all evidently part of a technical vocabulary of rhetoric established before he began to write: his purpose here is not to define these terms for the uninitiated, but to show how they may be fitted into the account of deductions in the figures.

68b15. The term *epagōgē* is traditionally translated 'induction' (from its Latin cognate *inductio*). There are strong reasons for preserving this tradition: 'induction' is a technical term with Aristotle, the sense of which must be determined from his use; it is historically important that *epagōgē* is the Greek ancestor of 'induction'; and the term is deeply entrenched in the secondary literature. With this said, the reader should beware of reading associations from the subsequent history of philosophy into Aristotle's word, at least without careful consideration of what he says. The correct sense would be better captured by 'leading up,' or possibly 'introduction.' Ross's survey of the uses of *epagōgē* (481–484) is still the best.

It is a bit surprising to find Aristotle speaking of a 'deduction from induction' (*sullogismos ex epagōgēs*). Elsewhere, he presents deduction and induction as the two possible types of argument and sharply separates them (see A 25, 42a3–4 and *Topics* I.12, VIII.2; *Rhetoric* I.2, 1356a35–b11). But the overall project of these latter sections of Book B is the assimilation of 'absolutely any form of conviction whatever' to the theory of figured deductions: now we see just how far Aristotle intends to carry this. Ross calls B 23 a '*tour de force* in which A. tries at all costs to bring induction into the form of syllogism,' (486) and says (I think correctly) that it was produced by an Aristotle 'filled with enthusiasm for his new-found discovery of the syllogism' (50).

68b15–27. Aristotle's argument here presupposes that *epagōgē* is a process of bringing forward each of a number of individual cases and establishing the same thing about each. He describes this as a sort of proof of *AaB* from *AaC* and *BaC* together with the premise that *only C* is *B*; therefore, *B* and *C* convert, and by the result established at B 22, 68a16–25, *AaB*. The process of induction must be thought of, not as the inference to *AaB*, but as the process of bringing up the cases one by one (thus, in 68b15, Aristotle distinguishes between the deduction from *epagōgē* and *epagōgē* itself: the deduction takes place *after* the induction has established *AaC*).

The requirement that induction deal with all the particulars or all the cases is probably one Aristotle inherited from the rhetorical tradition, rather than one of his own suggestions. Here, he takes it for granted that 'induction is through them all' and tries to show that under that assumption, the inference is sound. On a related point, the term *C* in his example actually does not correspond to any predicate: it is, rather, 'composed of every one of the particulars.'

68b21–22. The phrase in parentheses seems to assume what is to be proved, i.e., that *AaB*. Ross, noting that one manuscript reads 'for every bileless thing C is long-lived,' suggests 'for every C is long-lived,' while Tredennick rejects the phrase. Neither of these is very satisfying (there is no textual support for the rejection, and Ross's proposal gives us a somewhat unusual construction *pan gar to acholon G* where we would normally expect Aristotle to say *pan gar to acholon to G*). Tredennick may be right, but I offer the following speculation: Aristotle does not say that C comprises all of the *long-lived* things, but all of the *bileless* things. As a result, the *epagōgē* in this case must operate by considering, one after another, each of the *bileless* things, every one of which then is found to be long-lived (by observation) and known to be bileless (by selection). Thus, what Aristotle is saying with the troublesome phrase is this: since, as a matter of fact, everything bileless is long-lived, it will result that in selecting bileless things for consideration we are also selecting long-lived things. When we have exhausted the entire class of bileless things (so that we know that B does not 'extend beyond' C but converts with it), we are in a position to infer that whatever is bileless is long-lived. This interpretation is also supported by the reading of manuscript *n*, which has 'the particular long-lived *things*' (plural) at 68b20.

A long tradition faults Aristotle's apparent supposition here that induction must rely on a study of *all* individual cases (so-called 'perfect induction'). But we can interpret 'all cases' in two ways: either as an examination of every *individual* falling under a certain predicate or as an examination of every separate *kind* falling under it. Aristotle does not tell us explicitly which he has in mind, but his biological example lends itself best to the latter interpretation (it would, accordingly, be every bileless *species*, not every individual bileless animal, that we would have to include in our survey, and this is not an unreasonable requirement). Later, in B 24, we read that *epagōgē* proves from 'all the individuals' (69a17: *ex hapantōn tōn atomōn*). But Aristotle frequently uses *atomon* to mean 'species having no subspecies' (cf. *Posterior Analytics* I.23, 84b15; II.5, 91b32).

68b24–27. 'it has been proved earlier': at 68a16–25. The 'extreme' here is the term to which the 'same things' belong.

68b30–37. Aristotle here links up his assimilation of *epagōgē* to the figure-theory with his theory of the cognition of indemonstrable first principles through induction in *Posterior Analytics* II.19, at the same time getting at least a sort of characterization in the same terms of his customary distinction between what is more familiar 'to us' and what is more familiar 'in itself' or 'by nature' (for which see, among other passages, *Posterior Analytics* I.2, 71b33–72a5; *Physics* I.1, 184a16–25; *Metaphysics* VII.3, 1029b3–12; *Nicomachean Ethics* VI.3, 1139b34–36). Since the account of *Posterior Analytics* II does not assume anything like experience with every particular, some details must be supplied to bring the two accounts into harmony; this is a thorny problem which I cannot address in any useful fashion in these Notes. (See Hamlyn *1976*, Engberg-Pederson *1979*, McKirahan *1983*.)

Chapter 24

68b38–69a19. Reasoning by example as Aristotle here presents it has the following structure: we wish to prove that *AaC* and do so by first taking *D*, which is like *C* in some way and where it is familiar that *AaD*. We now take another predicate *B* such that both *BaC* and *BaD*; we 'infer' *AaB* from *AaD* and *BaD* (and perhaps other examples as well); and finally we deduce *AaC* from *AaB* and *BaC*.

The critical point here is the inference from *BaD* and *AaD* to the generalization *AaB*. This, in fact, is much closer to the modern sense of the term 'induction' (inferring a generalization from one of its cases) than the process Aristotle defines as *epagōgē*, the more so since Aristotle allows that several examples might be used. However, he indicates two points of distinction between example and induction: (1) reasoning by example does not consider all cases, (2) reasoning by example 'connects the deduction to the extreme' while *epagōgē* does not. We can make sense of this if we suppose that *epagōgē* is used to find demonstrations. It will follow that in establishing *AaB* by *epagōgē*, where *C* is the class of things selected because they are *B* and then observed to be *A*, our ultimate purpose is to *deduce* the conclusion *AaC* from the premises *AaB*, *BaC*. In the case of an example, however, the purpose is to establish what amounts to another particular case: we want to prove that war with the neighboring Thebans would be evil for the Athenians, and so we first offer a familiar example (the war of the Thebans with their neighbors the Phocians was an evil for them) to establish the principle 'war with one's neighbors is an evil' and then apply this to the particular case at hand.

Aristotle's example of an example is one of the few passages in the *Analytics* for which an absolute date (after 353 B.C.) has been suggested on historical grounds (see Ross 22, 488).

69a14–19. The part-whole relations Aristotle mentions here are all relations of 'middle term' (i.e., 'term that does not appear in the conclusion') to 'minor extreme' (predicate of the conclusion). In a straightforward deduction in Aristotle's standard case (*Barbara*), the middle term is to the minor as whole to part; in induction, according to the analysis of B 23, it is as part to whole. In an argument through example, however, it is logically coordinate with the minor extreme, since both are parts of (fall under) the major term but neither is part of the other. The sense in which argument through induction 'proves the extreme to belong to the middle from all the individuals' is spelled out in B 23, 68b15–29, where it is the *major* extreme that is so proved to belong. (On 'individuals' in 69a17 see the notes above on B 23.)

To 'connect' (*sunhaptein*), as Aristotle uses the term here, is to link up two terms by means of a middle term; B 23 emphasizes the fact that *epagōgē* does not do this in order to show its role in establishing unmiddled premises. On the account just given, an example is really a device for producing conviction in the major premise of a deduction, which is then, in turn, used to establish another conclusion. Thus, in order to establish that war with neighbors is an evil, we use the example of the war of the Thebans with the Phocians; having established this, we then use it as major premise to deduce the conclusion that war with the Thebans is an evil for the Athenians. Since there is a deduction, there is a middle term which 'connects' two extremes.

One minor puzzle remains. Aristotle says that *epagōgē* does not, and *paradeigma* does, connect the *deduction* with the 'extreme.' This may simply be a loose usage, but it is just possible that *sullogismos* here means 'minor term';

Aristotle sometimes uses *sullogismos* to refer to the conclusion (*sumperasma*) of a deduction, and he sometimes uses *sumperasma* to mean 'minor term' (e.g., *Posterior Analytics* I.11, 77a21).

Chapter 25

69a20–36. The term *apagōgē* is as difficult to translate as its cousin *epagōgē*. The traditional English rendering is 'reduction,' but this invites confusion with *anagōgē*. The process Aristotle defines involves leading the argument away from one question or problem to another more readily resolved (thus I opt for 'leading away').

As defined here, *apagōgē* is a matter of finding premises from which something may be proved. Aristotle says that this constitutes *apagōgē* under two circumstances: (1) the major premise is 'clear' and the minor premise is at least as convincing as the conclusion; (2) there are 'fewer middles' of the minor premise than of the conclusion. Criterion (1) is epistemic, while (2) is proof-theoretic: both are included in *Posterior Analytics* I.2 among the requirements which the premises of a demonstration must satisfy. For this reason, I have translated *epistēmē* here as 'scientific understanding,' the technical sense it carries in that treatise. (In the example which follows immediately I revert to the briefer 'science.')

69a28–29. 'for it is closer . . . did not have scientific understanding': the text here presents some problems. Most manuscripts give the text as *enguteron gar tou epistasthai dia to proseilēphenai tēn AG epistēmēn proteron ouk echontas*. Ross puts a comma after 'science' (*epistēmēn*), which would give the sense 'for it is closer to scientific understanding because of taking in addition the knowledge of AC, which we previously did not have.' Since the entire point is to *establish* knowledge of AC here, this seems absurd; accordingly, Ross changes 'AC' to 'AB.' The sentence would then say that we get closer to a scientific understanding of AC ('Justice is teachable') by 'taking the knowledge' of AB ('Justice is a science'). But this does not seem to illustrate what Aristotle has just said. The definition of *apagōgē* supposes that 'the first clearly belongs to the middle': we may, I think, take this to mean that the major premise AB is scientifically known. However, neither the minor premise BC nor the conclusion AC are known. Under those circumstances, if the minor premise BC is 'more convincing' than the conclusion AC, then assuming *that minor premise* brings us closer to scientific understanding because the assumption made is more convincing. On Ross's interpretation, we must assume AB, but Aristotle presents this premise as already 'clear' (and thus there is no need to 'assume' it). It is also unclear to me how we get 'closer' to science on Ross's view: closer than what? My translation keeps the text as it is, but punctuates after *proseilēphenai* ('taking in addition'). Pacius' text, *proseilēphenai tēi AG tēn BG* ('assuming BC in addition to AC'), would support essentially the same interpretation; Waitz adopts this, and Tricot and Rolfes give different renderings of it. However, this has less manuscript authority and does not give a good sense (it makes Aristotle appear to treat the conclusion to be proved as a sort of assumption).

69a35–36. 'when BC is unmiddled': Aristotle supposes that the major premise AB is 'evident,' and takes this, in turn, to mean both 'without a middle' (*amesos*) and 'familiar in itself.' If, in addition, the minor premise also is without a middle, then both premises of the deduction are indemonstrable first principles, and therefore the deduction is a demonstration producing scientific understanding.

Chapter 26

The subject of objections (*enstaseis*) is treated in *Rhetoric* II.25 and mentioned in the *Topics*, *Sophistical Refutations*, and *Posterior Analytics*: evidently, this was a technical term of rhetorical theory before Aristotle. (*Posterior Analytics* I.4, 73a32–34, I.6, 74b18–21, and I.12, 77b34–39 are worth comparing here.) An objection in this sense is an attack on one of the *premises* of an argument: attacking an argument by giving a counterargument against its conclusion is 'refutation' (*elenchos*: see B 20) or 'counterdeduction' (*antisullogismos*: see *Topics* VIII.8). Although Aristotle's initial definition suggests that an objection is simply a statement either contrary or contradictory to a premise, the evidence of the *Rhetoric* and later passages in the present Chapter indicate that what Aristotle is talking about is something comparable to an 'enthymeme': a statement such that either the contrary or the contradictory of the premise objected to follows from it with another obvious but unstated premise.

69a39–b1. The statement that a premise 'either cannot be particular at all, or not in universal deductions' is puzzling: it is, of course, true that a universal deduction must have only universal premises, but it is quite opaque how this differentiates objections from premises in an interesting way (an objection also can only be particular *in objection to* a universal premise).

69b1–8. The 'two ways' are clear enough, but the restriction to two figures is more difficult to understand: this is one of the reasons Ross concludes that B 26 'suffers from compression and haste.' Aristotle assumes that if the objection is universal in form it must be deduced in the first figure, while if it is particular it must be deduced in the third. Ross explains this by supposing that the objection is actually brought against a premise which is itself already the conclusion of a deduction and that, for some reason, Aristotle requires the objection to give a deduction in the same figure as that original deduction: otherwise, he says, the explanation 'opposites are concluded only in the first and third figures' makes no sense. However, he himself points out that on this interpretation the reference to the third figure makes no sense either, since contradictory statements cannot be deduced in that figure.

It may be impossible to straighten out Aristotle's thought in this Chapter, but it seems to me relevant that his discussion revolves, to some extent, around the problem of *finding* an objection. The objection found is a premise which shares one term with the premise objected to, and has a new term in a relationship to it determined by the type of premise objected to and the type of objection (contrary or contradictory) desired. Looking for a premise in this way recalls the procedures of A 27–28 (44a11–35); and there, Aristotle found premises for universal conclusions in the first figure and for particular conclusions in the third. The only exceptions are second-figure *Camestres* and fourth-figure *Bramantip* as alternatives for *e* and *i* conclusions respectively; and Aristotle says that the first requires a 'prior deduction' (*ek prosullogismou*, 44a22–23) and that the second is a 'converted deduction' (*antestrammenos sullogismos*, 44a31). He thus assigned a special priority to the four deductions *Barbara*, *Celarent*, *Darapti*, *Felapton*, as the ones to which one should look in searching for premises. In view of his further requirement that it should be immediately evident that the contrary or the contradictory of the initial premise follows from the objection stated (cf. 69b32–36 below), the remarks in A 28 about *Camestres* and *Bramantip* probably disqualify them here.

69b8–19. In these examples, the objection itself is really the *major premise* of a deduction from which the contradictory or contrary of the premise attacked follows. The minor premise is supposed to be something obvious, and therefore unstated. For an affirmative premise, Aristotle's two cases are: (1) objecting contrarily, we find a term *C* such that *CaB* is obvious and object that *AeC* (and *AeB* follows in *Celarent*); (2) objecting contradictorily, we find a term *C* such that *BaC* is obvious and object that *AeC* (*AoB* then follows in *Felapton*). For a negative premise, the cases are identical except that the objection itself (i.e., the major premise) is of the form *AaB*.

69b19–28. Though Aristotle's language here is highly compressed, and as a result obscure, his point is clear enough. The objection sought is to be a premise having as one term the *predicate* of the premise attacked; Aristotle defines its relationship to the other premise, which depends on the nature of the objection. The way in which Aristotle describes the procedure comports well with the methods of A 28: from among those which the predicate belongs to all of (or to none of), we search for a term which belongs to all of the subject (or to which the subject belongs to all of).

69b28–32. 'Those premises': this may give a clue as to why Aristotle considers only first- and third-figure arguments for objections: the two procedures he has defined (one for universal and one for particular conclusions) can be applied both to affirmative and to negative premises simply by changing the major premise sought from *e* to *a*.

69b32–36. 'And in addition': This suggests that Aristotle avoids the second figure partly because he regards it as not yielding sufficiently obvious deductions. The phrase 'because C does not follow it' is ambiguous: 'it' could be either A or B. On the first possibility, the case is objecting to *AaB* by stating *CeA* (where *CaB* is evident); on the second, the objection is *CeB* and the unstated premise is *CaA*. Ross opts for the first interpretation on the grounds that it is in this way less obvious what the unstated premise must be. But in A 28 the second-figure deduction in *Camestres* (which corresponds here to the second case) is said to require additional argument.

69b35–36. 'turn aside': Aristotle uses the verb *ektrepesthai* of pursuing a wrong approach to a problem or a wrong line of inquiry (cf. *Physics* I.8, 191a26, 191b32.

69b36–37. Ross, following Susemihl, rejects this sentence as out of place and inconsistent with the contents of B 27.

69b38–70a2. This closing note attests to the unfinished state of B 26. The commentators note that these other types of objections correspond to the last three of the four enumerated in *Rhetoric* II.25; the first of the four, 'from the thing itself' (*aph' heautou*), corresponds closely to the type of objection treated in B 26, as Ross notes. Since the fourth type of objection is described as 'judgments on the part of famous men' (*hai kriseis hai apo gnōrimōn andrōn*), I have translated *kata doxan* as 'according to reputation' rather than 'according to opinion.'

Chapter 27

70a3. Ross, noting that the Chapter begins somewhat abruptly and, unlike B 23–26, without a summary definition of its subject, transposes a sentence here from 70a10. But despite what Ross says, the subject of the section is not enthymemes (the word *enthumēma* occurs only once in it), but signs and likelihoods. This is confirmed by the references to it from the *Rhetoric* (I.2, 1357b21–25; II.25, 1403a4–5, 1403a10–12), which all concern signs and likelihoods.

The distinction between a likelihood (*eikos*) and a sign (*sēmeion*) is that they are defined by different sorts of criteria: a premise is a likelihood if it is 'well-known' or 'accepted' (*endoxos*), which, as the *Topics* makes clear, is a matter of the attitudes of belief people have towards it. A sign, by contrast, is defined as such by its role in a kind of deduction. As with Aristotle's other definitions, this is not intended to explain the term for those (like us) who are ignorant of its meaning, but to accommodate it in the deductive theory of the figures. The examples later in the Chapter make it clear enough what a sign is: it is just a statement predicating *A* of *B*, offered in support of the claim that *C* is predicated of *B*. For instance, wishing to establish the statement 'This woman is pregnant,' one might say, 'This woman has milk.'

70a5–6. Note that the examples here concern conduct typical of people who have certain emotional attitudes towards others and provide a basis for inferring those attitudes: we infer that X loves Y because X shows affection for Y, we infer that X is envious of Y because X hates (expresses hatred for) Y. I have slightly expanded the translations to reflect this.

70a9–10. 'Enthymemes' (*enthumēmata*) are discussed at length in the *Rhetoric*. The term is a difficult one to render without prejudice, meaning something like 'reason' or 'thing that makes one believe.' However, since it plays no important role in the *Prior Analytics*, its full elucidation may gratefully be foregone here.

70a11–24. The main point of the Chapter is to classify all occurrences of signs in terms of the figures. Aristotle distinguishes three cases: (1) we offer *BaC* as a sign for *AaC* when it is true that *AaB*; (2) we offer *AaC* as a sign for *AaB* when it is true that *BaC*; (3) we offer *AaC* as a sign for *BaC* when it is true that *AaB*. In case (1), the conclusion actually follows from the sign and the (unstated) other premise. In (2), the conclusion does not follow, although *AiB* does, and in any event, the argument resembles the pattern in *epagōgē*. In (3), neither the conclusion nor anything else follows from the premises. All three examples that he offers are plausible as rhetorical or forensic arguments; Aristotle is presumably trying to show that even though (3) is widely used, it should not be.

70a24–28. A sign is a stated premise with an unstated partner: if we actually state the second premise, we are not offering a sign but giving a deduction. (But as we see in the next passage, it is not really a deduction in cases (2) and (3).)

70a28–38. The crucial difference between the three cases is their deductive status. 'Nonbinding' (*lutos*, *lusimos*) means 'refutable' (cf. the use of the verb *luein* in *Metaphysics* XII.10, 1075a31, 33). Aristotle is saying that if the premises of a sign of type (1) are true, then no objection can be brought against it, since it is a proof, whereas even if the premises in cases of types (2) and (3) are true the conclusion may fail to be and so may possibly be refuted. 'The truth, then, can occur in all signs' probably means only that in each case the conclusion *may* be true when the premises are.

70b1–6. Aristotle ponders two alternative ways of distinguishing between signs and 'evidences' (*tekmēria*). The latter is a legal term, amounting to 'proofs' (a *tekmērion* constitutes conclusive evidence for something, while a *sēmeion* is merely an indication). The 'middle' here is the sign in case (1) above: the actual sign *is* the middle term, as having milk is a sign of being pregnant. In the other cases, the sign in this sense is not the middle term for premises in the relevant figure. Aristotle here wonders whether it is the term itself, or the entire deduction, which should be called 'sign' or 'evidence.'

70b7–38. This terminal section, on 'recognizing natures' (*phusiognōmonein*), seems to have no very close connection with the *Prior Analytics*. It might have been included (or appended by an editor) because this skill also formed part of the orator's bag of tricks. However, the discussion never refers to the other themes of the *Prior Analytics*. It seems more likely, therefore, that this passage found its place here only because it is concerned with certain physical states as *signs* of states of character. The art of physiognomonics was evidently established before Aristotle's time, in the fifth century: Alexander of Aphrodisias (*De Fato* 6) recounts an anecdote of an encounter between Socrates and the physiognomonist Zopyrus. A pseudo-Aristotelian (but probably Peripatetic) treatise with the title *Physiognomonics* has come down to us. As both this passage and that treatise make clear, the gist of this 'art' was a system of associations between anatomical characteristics and traits of character, based in large part on purported associations found in animals.

70b11. 'passions' (*orgai*): the word *orgē* in Aristotle (or generally in Greek) usually means 'anger,' but it also can mean 'strong emotion'; the use of the plural here, which is comparatively uncommon in Aristotle, together with the context, make 'passions' a good English version (cf. Tricot's French 'passions').

70b11–31. Aristotle concedes that physiognomonics could indeed work under certain restricted circumstances, to wit: (1) natural affections (*phusika pathēmata*) 'change' body and soul together; (2) each condition has one and only one sign; (3) we can determine both the affection and the sign peculiar to each animal species. The reasoning then proceeds as follows: If there is some affection which naturally belongs to all animals of a certain species (but only incidentally to animals of other species), then by (1) there must be some unique bodily sign associated with this affection; and by (2), this must be the unique sign associated with that affection wherever it is found. Aristotle does not appear to have any quarrel with (1), which evidently rests on the observation that there are naturally determined 'signs' (expressions) of various emotions. He also does not question (2), though his language suggests that he is merely accepting it for the sake of argument. What he does, instead, is raise difficulties about making use of (3) in those cases in which a species has more than one natural trait. The suggested solution, a sort of method of differences, is straightforward enough.

70b19. 'affection' (*pathos*): Ross notes that here this would have to denote the 'sign,' i.e., the bodily characteristic, rather than the affection of character (which is what *pathos* means throughout the rest of the passage); accordingly, he rejects this occurrence of the word. But Aristotle is perfectly capable of such a switch in word meaning within a passage: compare A 43 and the associated Note.

70b21. 'a man': Ross brackets the article before *anthrōpos*, evidently because he thinks the sense would otherwise have to be 'man in general,' i.e., every man without exception. But in connection with *estai*, 'is possible,' this is not very different from 'some men are brave,' which is what Aristotle means here.

70b32–38. This note offers a schema for physiognomonic explanations using the theory of figured deductions, in the style common throughout B. There is nothing here that Aristotle might not have said, but also nothing beyond an obvious application of Aristotle's techniques to the case in question.

APPENDIX I
A LIST OF THE DEDUCTIVE FORMS IN *PRIOR ANALYTICS* A 4–22

I list below the various premise pairs which Aristotle shows to yield a conclusion in A 4–22. Each deduction or premise-pair is accompanied by a textual citation and a brief indication of the way in which Aristotle proves the existence or nonexistence of a deduction for that case. As a convenience, I include the traditional mood names for premise pairs which correspond to them. Some modal-premise combinations corresponding to these pairs do not yield conclusions, and some of the modal pairs which yield conclusions do not correspond to named pairs. Question marks indicate points in which Aristotle's text is difficult to interpret. Readers who wish more detailed accounts should consult the table given by Ross and the studies of Becker, McCall, and Wieland.

The largest part of list is taken up with modally qualified forms. Here, I use **A** to indicate 'assertoric' premises (those with no modal qualifier). In discussing combinations of one assertoric and one necessary premise, Aristotle frequently takes it for granted that an assertoric conclusion can be deduced and only proves that a necessary conclusion cannot be. I indicate these by noting that he gives a 'proof that *not* **N**.' 'Modal conversion' means the conversion of a possible affirmative premise into its corresponding negative, or vice versa. Many of Aristotle's proofs rely on the use of conversion to bring about a previously established modal deduction. I indicate this by giving the figure and the modal qualifications of the premises. Thus, 'conversion to I **NA**' means 'conversion leading to a first-figure premise pair with necessary major premise and assertoric minor.' Aristotle often says that a given proof proceeds 'likewise' or 'as before,' and it is sometimes quite unclear what he intends. I have left many of these remarks unresolved.

The traditional names for the incomplete forms actually encode instructions for carrying out proofs. The first letter of the name (B, C, D, F) indicates the first-figure form to which the proof appeals; 's' following a vowel indicates that the corresponding premise (always an *e* or *i*) is to be converted (*conversio* *simplex*); 'p' following 'a' indicates 'conversion by limitation' (*conversio per accidens*) of a universal premise, i. e., conversion into a particular premise (*a* into *i*, *e* into *o*); 'c' indicates proof through impossibility; and 'm' indicates that the premises must be interchanged. (Other letters, such as 'l' and 'n,' have no significance.) Thus, the name *Camestres* tells us that a proof that an *e* conclusion follows from an *a* major premise and an *e* minor may be contructed by converting the first premise (*Camestres*) and interchanging the premises (*Camestres*), giving the first-figure form *Celarent* (*Camestres*), then converting the conclusion (*Camestres*). *Bocardo* indicates that an *o* conclusion can be proved to follow by assuming the contradictory of the proposed conclusion (an *a* sentence), giving the premises of *Barbara* (*Bocardo*), from which the contradictory of the first premise (*Bocardo*) follows.

NONMODAL (ASSERTORIC) DEDUCTIONS

FIGURE I (A 4)		
AaB, *BaC* ⊢ *AaC*	*Barbara*	complete (25b37–40)
AeB, *BaC* ⊢ *AeC*	*Celarent*	complete (25b40–26a2)
AaB, *BiC* ⊢ *AiC*	*Darii*	complete (26a23–25)
AeB, *BiC* ⊢ *AoC*	*Ferio*	complete (26a25–27)
FIGURE II (A 5)		
MeN, *MaX* ⊢ *NeX*	*Cesare*	conversion (27a5–9)
MaN, *MeX* ⊢ *NeX*	*Camestres*	conversion, impossibility (27a9–15)
MeN, *MiX* ⊢ *NoX*	*Festino*	conversion (27a32–36)
MaN, *MoX* ⊢ *NoX*	*Baroco*	impossibility (27a36–b3)
FIGURE III (A 6)		
PaS, *RaS* ⊢ *PiR*	*Darapti*	conversion, impossibility, *ekthesis* (28a17–26)
PeS, *RaS* ⊢ *PoR*	*Felapton*	conversion, impossibility (28a26–30)
PaS, *RiS* ⊢ *PiR*	*Datisi*	conversion (28b7–11)
PiS, *RaS* ⊢ *PiR*	*Disamis*	conversion, impossibility, *ekthesis* (28b11–15)
PoS, *RaS* ⊢ *PoR*	*Bocardo*	impossibility, *ekthesis* (28b17–21)
PeS, *RiS* ⊢ *PoR*	*Ferison*	conversion (28b33–35)

DEDUCTIONS WITH ONE OR MORE MODALLY QUALIFIED PREMISES

NN: (A 8: All forms except *Baroco* and *Bocardo*, 29b36–30a5)

FIGURE I		
N*AaB*, N*BaC* ⊢ N*AaC*	*Barbara*	[complete]
N*AeB*, N*BaC* ⊢ N*AeC*	*Celarent*	[complete]
N*AaB*, N*BiC* ⊢ N*AiC*	*Darii*	[complete]
N*AeB*, N*BiC* ⊢ N*AoC*	*Ferio*	[complete]

FIGURE II

$NAeB, NAaC \vdash NBeC$	*Cesare*	conversion
$NAaB, NAeC \vdash NBeC$	*Camestres*	conversion
$NAeB, NAiC \vdash NBoC$	*Festino*	conversion
$NAaB, NAoC \vdash NBoC$	*Baroco*	*ekthesis* (30a6–14)

FIGURE III

$NAaC, NBaC \vdash NAiB$	*Darapti*	conversion
$NAeC, NBaC \vdash NAoB$	*Felapton*	conversion
$NAiC, NBaC \vdash NAiB$	*Disamis*	conversion
$NAaC, NBiC \vdash NAiB$	*Datisi*	conversion
$NAoC, NBaC \vdash NAoB$	*Bocardo*	*ekthesis* (30a6–14)
$NAeC, NBiC \vdash NAoB$	*Ferison*	conversion

N + A:

FIGURE I (A 9)

$NAaB, BaC \vdash NAaC$	*Barbara*	*ekthesis* (30a17–23)
$NAeB, BaC \vdash NAeC$	*Celarent*	*ekthesis* (30a17–23)
$AaB, NBaC \vdash AaC$	*Barbara*	proof that *not* N through impossibility and through terms (30a23–33)
$AeB, NBaC \vdash AeC$	*Celarent*	proof that *not* N through impossibility and through terms (30a23–33)
$NAaB, BiC \vdash NAiC$	*Darii*	*ekthesis* (30a37–b2)
$NAeB, BiC \vdash NAoC$	*Ferio*	*ekthesis* (30a37–b2)
$AaB, NBiC \vdash AiC$	*Darii*	proof through terms that *not* N (30b1–6)
$AeB, NBiC \vdash AoC$	*Ferio*	proof through terms that *not* N (30b1–6)

FIGURE II (A 10)

$NAeB, AaC \vdash NBeC$	*Cesare*	conversion to I NA (30b9–13)
$AaB, NAeC \vdash NBeC$	*Camestres*	conversion to I NA (30b14–18)
$AeB, NAaC \vdash BeC$	*Cesare*	[no argument]
$NAaB, AeC \vdash \mathrm{BeC}$	*Camestres*	conversion to I AN; argument that *not* N through impossibility, terms (30b20–40)
$NAeB, AiC \vdash NBoC$	*Festino*	conversion to I NA (31a5–10)
$NAaB, AoC \vdash BoC$	*Baroco*	that *not* N through terms (31a10–15)
$AeB, NAiC \vdash BoC$	*Festino*	that *not* N through terms (31a15–17)
$AaB, NAoC \vdash BoC$	*Baroco*	that *not* N through terms (31a15–17)

FIGURE III (A 11)

$NAaC, BaC \vdash NAiB$	*Darapti*	conversion to I NA + *ekthesis* (31a24–30)
$AaC, NBaC \vdash NAiB$	*Darapti*	conversion to I NA (31a31–33)

N*AeC*, *BaC* ⊢ N*AoB*	*Felapton*	conversion to I NA + *ekthesis* (31a33–37)
AeC, N*BaC* ⊢ *AoB*	*Felapton*	conversion to I AN; terms (31a37–b10)
AiC, N*BaC* ⊢ N*AiB*	*Disamis*	conversion to I NA + *ekthesis* (31b12–19)
N*AaC*, *BiC* ⊢ N*AiB*	*Datisi*	conversion to I NA + *ekthesis* (31b19–20)
AaC, N*BiC* ⊢ *AiB*	*Datisi*	conversion to I AN; terms (31b20–31)
N*AiC*, *BaC* ⊢ *AiB*	*Disamis*	that *not* N through terms (31b31–33)
N*AeC*, *BiC* ⊢ N*AoB*	*Ferison*	'same as before' (31b35–37)
AoC, N*BaC* ⊢ *AoB*	*Bocardo*	that *not N* through terms (31b40–32a1)
AeC, N*BiC* ⊢ *AoB*	*Ferison*	terms (32a1–4)
N*AoC*, *BaC* ⊢ *AoB*	*Bocardo*	terms (32a4–5)

PP:

FIGURE I (A 14)

P*AaB*, **P***BaC* ⊢ **P***AaC*	*Barbara*	complete (32b38–33a1)
P*AeB*, **P***BaC* ⊢ **P***AeC*	*Celarent*	complete (33a1–5)
P*AaB*, **P***BeC* ⊢ **P***AaC*		modal conversion (33a5–12)
P*AeB*, **P***BeC* ⊢ **P***AaC*		modal conversion (33a12–17)
P*AaB*, **P***BiC* ⊢ **P***AiC*	*Darii*	complete (33a23–25)
P*AeB*, **P***BiC* ⊢ **P***AoC*	*Ferio*	complete (33a25–27)
P*AaB*, **P***BoC* ⊢ **P***AiC*		modal conversion (33a27–34)
all *i*/*o*–*a*/*e* combinations		rejected by terms, conversion to previous cases (33a34–b17)

FIGURE II (A 17)

all forms rejected		failure of **P***e* conversion (36b35–37a31)
P*AeB*, **P***AaC*		rejected by several arguments, terms (37a32–b10)
all other forms		rejected by terms (37b10–16)

FIGURE III (A 20)

P*AaC*, **P***BaC* ⊢ **P***AiB*	*Darapti*	conversion (39a14–19)
P*AeC*, **P***BaC* ⊢ **P***AoB*	*Felapton*	conversion (39a19–23)
P*AeC*, **P***BeC* ⊢ **P***AiB*		modal conversion (39a26–28)
P*AaC*, **P***BiC* ⊢ **P***AiB*	*Datisi*	conversion (39a31–35)
P*AiC*, **P***BaC* ⊢ **P***AiB*	*Disamis*	conversion (39a35–36)
P*AeC*, **P***BiC* ⊢ **P***AoB*	*Ferison*	conversion (39a36–38)
P*AoC*, **P***BaC* ⊢ **P***AoB*	*Bocardo*	conversion (39a36–38)

P*AeC*, P*BoC* ⊢ P*AiB*		modal conversion (39a38–b2)
P*AoC*, P*BeC* ⊢ P*AiB*		modal conversion (39a38–b2)
all forms with two particulars		rejected by terms (39b2–6)

P + A:

FIGURE I (A 15)

P*AaB*, *BaC* ⊢ P*AaC*	*Barbara*	complete, *ekthesis* (33b33–36)
P*AeB*, *BaC* ⊢ P*AeC*	*Celarent*	complete (33b36–40)
AaB, P*BaC* ⊢ P(*AaC*)	*Barbara*	proof through impossibility (34a34–b2)
AeB, P*BaC* ⊢ P(*AeC*)	*Celarent*	proof through impossibility; terms (34b19–35a2)
AaB, P*BeC* ⊢ P(*AaC*)		modal conversion (35a3–11)
AeB, P*BeC* ⊢ P(*AaC*)		modal conversion (35a11–20)
all forms with minor *BeC*		rejected by terms (35a20–24)
P*AaB*, *BiC* ⊢ P*AiC*	*Darii*	complete (35a30–35)
P*AeB*, *BiC* ⊢ P*AoC*	*Ferio*	complete (35a30–35)
AaB, P*BiC* ⊢ P(*AiC*)	*Darii*	through impossibility (35a35–40)
AeB, P*BiC* ⊢ P(*AoC*)	*Ferio*	through impossibility (35a35–40)
AaB, P*BoC* ⊢ P(*AiC*)		modal conversion (35b5–8)
AeB, P*BoC* ⊢ P(*AoC*)		modal conversion (35b5–8)
all forms with minor *BoC*		rejected by terms (35b8–14)
all forms with two particulars		rejected by terms (35b14–22)

FIGURE II (A 18)

P*AeB*, *AaC*	(*Cesare*)	rejected 'as before' (37b19–23)
AaB, P*AeC*	(*Camestres*)	rejected 'as before' (37b19–23)
AeB, P*AaC* ⊢ P(*BeC*)	*Cesare*	conversion to I **AP** (37b24–28)
P*AaB*, *AeC* ⊢ P(*BeC*)	*Camestres*	[conversion] (37b29)
AeB, P*AeC* ⊢ P(*BeC*)		modal conversion (37b29–35)
P*AeB*, *AeC* ⊢ P(*BeC*)		modal conversion (37b29–35)
all forms with two affirmatives		rejected by terms (37b35–38)
P*AeB*, *AiC*	(*Festino*)	rejected 'as before' (37b39–38a2)
AaB, P*AoC*	(*Baroco*)	rejected 'as before' (37b39–38a2)
AeB, P*AiC* ⊢ P(*BoC*)	*Festino*	conversion (38a3–4)
AeB, P*AoC* ⊢ P(*BoC*)		modal conversion (38a4–7)

FIGURE III (A 21)

AaC, P*BaC* ⊢ P(*AiB*)	*Darapti*	conversion to I **AP** (39b10–16)
P*AaC*, *BaC* ⊢ P*AiB*	*Darapti*	conversion to I **PA** (39b16–17)
P*AeC*, *BaC* ⊢ P*AoB*	*Felapton*	conversion to I **PA** (39b17–22)
AeC, P*BaC* ⊢ P(*AoB*)	*Felapton*	conversion to I **AP** (39b17–22)
AaC, P*BeC* ⊢ P(*AiB*)		modal conversion (39b22–25)
AeC, P*BeC* ⊢ P(*AoB*)		modal conversion (39b22–25)

AiC, **P***BaC* ⊢ **P***AiB*	*Disamis*	conversion to **I PA** (39b26–31)
P*AaC*, *BiC* ⊢ **P***AiB*	*Datisi*	conversion to **I PA** (39b26–31)
P*AiC*, *BaC* ⊢ **P**(*AiB*)	*Disamis*	conversion to **I AP** (39b26–31)
AaC, **P***BiC* ⊢ **P**(*AiB*)	*Datisi*	conversion to **I AP** (39b26–31)
P*AeC*, *BiC* ⊢ **P***AoB*	*Ferison*	conversion to **I PA** (39b26–31)
AeC, **P***BiC* ⊢ **P**(*AoB*)	*Ferison*	conversion to **I AP** (39b26–31)
P*AoC*, *BaC* ⊢ **P**(*AoB*)	*Bocardo*	through impossibility (39b31–39)
all forms with two particulars		rejected through terms (40a1–3)

P + **N**:

FIGURE I (A 16)

N*AaB*, **P***BaC* ⊢ **P**(*AaC*)	*Barbara*	'as previous' (35b37–36a2: cf. 34a34–b2)
P*AaB*, **N***BaC* ⊢ **P***AaC*	*Barbara*	complete (36a2–7)
N*AeB*, **P***BaC* ⊢ *AeC*	*Celarent*	through impossibility (36a7–17)
P*AeB*, **N***BaC* ⊢ **P***AeC*	*Celarent*	complete; cannot be done through impossibility (36a17–24)
N*AaB*, **P***BeC* ⊢ **P**(*AaC*)		modal conversion (36a25–27)
N*AeB*, **P***BeC* ⊢ **P***AeC*		modal conversion (36a25–27)
P*AaB*, **N***BeC*		rejected by terms (36a27–31)
P*AeB*, **N***BeC*		rejected by terms (36a27–31)
N*AeB*, **P***BiC* ⊢ *AoC*	*Ferio*	through impossibility (36a34–39)
P*AeB*, **N***BiC* ⊢ **P***AoC*	*Ferio*	not **A**, proof 'as before' (36a39–b2)
N*AaB*, **P***BiC* ⊢ **P**(*AiC*)	*Darii*	not **A**, proof 'as before' (36a39–b2)
all forms with universal **P** minor		rejected by terms (36b3–7)
all forms with universal **N** minor		rejected by terms (36b7–12)
two particulars or indefinites		rejected by terms (36b12–18)
P*AaB*, **N***BiC* ⊢ **P***AiC*	*Darii*	'as before'
N*AaB*, **P***BoC* ⊢ **P**(*AiC*)		(35*b* 28–30)
N*AeB*, **P***BoC* ⊢ *AoC*		(35*b* 30–31)

FIGURE II (A19)

N*AeB*, **P***AaC* ⊢ **P**(*BeC*) ⊢ *BeC*	*Cesare*	conversion to **I NP** for **P**; through impossibility for **A** (38a16–25)
P*AaB*, **N***AeC* ⊢ **P**(*BeC*) ⊢ *BeC*	*Camestres*	'same as previous' (38a25–26)
P*AeB*, **N***AaC* ⊢ BeC	(*Cesare*)	rejected by terms, detailed argument (38a26–b4)
P*AaB*, **N***AeC*	(*Camestres*)	same as previous (38b4–5)
N*AeB*, **P***AeC* ⊢ *BeC*		modal conversion and conversion to **I NP** (38b8–12)
P*AeB*, **N***AeC* ⊢ *BeC*		'likewise' (38b12–13)
all *aa* forms		rejected by terms (38b13–23)

$\mathrm{N}AeB, \mathrm{P}AiC \vdash BoC$	*Festino*	'same as universal' (38b25–27)
$\mathrm{N}AaB, \mathrm{P}AoC$		rejected by terms (38b27–29)
all forms with two affirmatives		rejected 'as before' (38b29–31)
$\mathrm{N}AeB, \mathrm{P}AoC \vdash BoC$		modal conversion (38b31–35)
all forms with two particulars		rejected by terms (38b35–37)

FIGURE III (A 22)

$\mathrm{N}AaC, \mathrm{P}BaC \vdash \mathrm{P}(AiB)$	*Darapti*	conversion to I NP (40a12–16)
$\mathrm{P}AaC, \mathrm{N}BaC \vdash \mathrm{P}AiB$	*Darapti*	conversion to I PN (40a16–18)
$\mathrm{P}AeC, \mathrm{N}BaC \vdash \mathrm{P}AoB$	*Felapton*	conversion to I PN (40a18–25)
$\mathrm{N}AeC, \mathrm{P}(BaC) \vdash AoB$	*Felapton*	conversion to I NP (40a25–32)
$\mathrm{N}AaC, \mathrm{P}BeC \vdash \mathrm{P}(AiB)$		modal conversion (40a33–35)
$\mathrm{P}AaC, \mathrm{N}BeC$		rejected by terms (40a35–38)
$\mathrm{N}AiC, \mathrm{P}BaC \vdash \mathrm{P}AiB$	*Disamis*	conversion to I PN (40a40–b3)
$\mathrm{P}AaC, \mathrm{N}BiC \vdash \mathrm{P}AiB$	*Datisi*	conversion to I PN (40a40–b3)
$\mathrm{P}AiC, \mathrm{N}BaC \vdash \mathrm{P}(AiB)$	*Disamis*	conversion to I NP (40a40–b3)
$\mathrm{N}AaC, \mathrm{P}BiC \vdash \mathrm{P}(AiB)$	*Datisi*	conversion to I NP (40a40–b3)
$\mathrm{P}AeC, \mathrm{N}BiC \vdash \mathrm{P}AoB$	*Ferison*	conversion to I PN (40a40–b3)
$\mathrm{P}AoC, \mathrm{N}BaC \vdash \mathrm{P}(AoB)$	*Bocardo*	through impossibility? (40a40–b3)
$\mathrm{N}AoC, \mathrm{P}BaC \vdash AoB$	*Bocardo*	through impossibility? (40b3–8)
$\mathrm{N}AeC, \mathrm{P}BiC \vdash AoB$	*Ferison*	conversion to I NP (40b3–8)
$\mathrm{N}AiC, \mathrm{P}BeC \vdash \mathrm{P}AiB$		modal conversion (40b8–11)
$\mathrm{P}AiC, \mathrm{N}BeC$		rejected 'as before' (40b10–12)